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American University of Sharjah
Department of Mathematics and Statistics

MTH 101 - Mathematics for Business I

Exam 1

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Time of Regular Class: 11:00 - 11:50

No questions are allowed during the exam, graphing calculators and cell phones are not allowed. This exam has 7 questions.

Good Luck.

1. (12 points) For a certain product you have

Price-Demand $p(x) = 100 - 5x$

Cost function $C(x) = 200 + 15x$

where x is in thousand of items.

a. Find the Revenue function $R(x)$ and the Profit function $P(x)$

$R(x) = \text{Price-demand} \times x$

$R(x) = 100x - 5x^2$

$P(x) = R(x) - C(x)$

$= 100x - 5x^2 - 200 - 15x$

$P(x) = 85x - 5x^2 - 200$

b. Find the vertex of the profit function

$a = -5, b = 85, c = -200$

* The vertex of the profit function $= \frac{-b}{2a}$

$= \frac{-85}{2 \times -5} = 8.5$ thousand of items

$P(8.5) = (85 \times 8.5) - (5 \times (8.5)^2) - 200$

$= 161.25$ thousand of dollar

* the vertex point
 items money
 $(8.5, 161.25)$

161.25
 8.5

c. Calculate the break even points

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-85 \pm \sqrt{(85)^2 - (4 \times -5 \times -200)}}{2 \times -5}$

$x_1 = 2.821$

$x_2 = 14.18$

① $(2.821, 242.315)$

② $(14.18, 412.7)$

$C(2.821) = 200 + (15 \times 2.821)$

$= 242.315$

$= 200 + (15 \times 14.18) = 412.7$

d. What is the wholesale price per item that produces the maximum profit?

$\frac{161.25}{8.5}$

$= 18.97$

~~$\frac{161.25}{8.5} = 18.97$ dollars~~

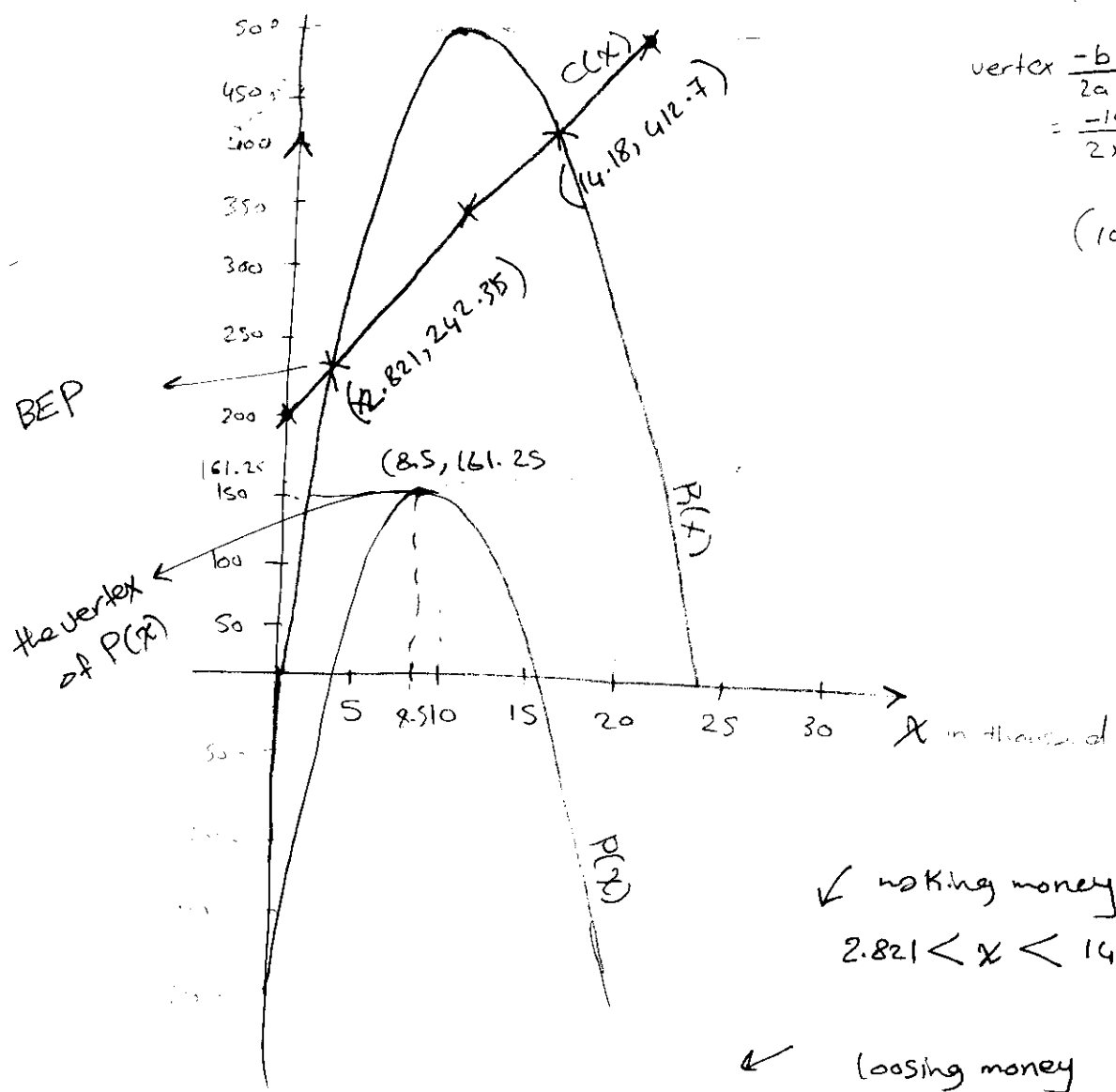
~~Price-demand $p(8.5) = 100 - (5 \times 8.5)$
 $= 57.5$ dollar~~

e. Sketch the graph of $R(x)$, $P(x)$ and $C(x)$. Indicate on your graph the following elements:

- i. the vertex of $P(x)$
- ii. the break even points
- iii. the interval(s) on the x -axis corresponding to a loss
- iv. the interval(s) on the x -axis corresponding to a profit

$C(x) = 200 + 15x$
 $(0, 200)$ $(20, 500)$

$R(x) = 100x - 5x^2$
 y-intercept $x=0$
 $y=0$ $(0,0)$
 vertex $\frac{-b}{2a}$
 $= \frac{-100}{2 \times -5} = 10$
 500
 $(10, 500)$



↙ making money
 $2.821 < x < 14.18$

↙ losing money

$0 \leq x < 2.821$
 $14.18 < x < 25$

~~2.821 < x < 14.18~~

~~$x > 14.18$~~

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2. (4 points) National Bank of Um-Al Quwain offers guaranteed deposits at 6.0% compounded semi-annually while the The National Bank Of Dubai offers 5.8% compounded daily. Which bank offers the best investment? Justify your answer. (Use a 365-day year).

1 UMAQUAIN

$$APY = \left[1 + \frac{0.06}{2} \right]^2 - 1 \times 100$$

$$= 0.0609 = \%6.09$$

2 DUBAI

$$APY = \left[1 + \frac{0.058}{365} \right]^{365} - 1 \times 100$$

$$= \%5.94$$

The National Bank of Um-Al Quwain is better because its interest is higher than the National Bank of Dubai which that will benefit me more.

3. (6 points) Sakina buys for \$20,000 of stocks. After 90 days, she sells her stocks for \$20,600. Unfortunately, for each of these transactions (buy and sell) she has to pay a commission of \$50 to the broker. Calculate the annual rate of simple interest Sakina earns on her investment. (Use a 360-day year).

Step 1

	20,000	\$ 20,000
		\$ 20,600
	20,000 + 50 = 20,050	

buy	20,000	20,050
sell	20,600	20,650

$$I = Prt$$

$$\frac{600}{20050} = 20050 \times r \times \frac{90}{360} \quad 5012 \frac{1}{2}$$

$$r = 0.1197$$

$$r = \%11.97$$

4. (6 points) A salesperson will be paid in a given month a salary which depends on the number of items he sells:

Money received	Items sold
\$1000	for the first 300 items or less
\$5 per item	for each of the next 200 items
\$10 per item	for each of the next 100 items
\$15 per item	for each of the items in excess over 600 items

Write a piecewise functions $S(x)$ for the salary of a salesperson selling x items in a given month.

$$S(x) = \begin{cases} 1000 & , 0 \leq x \leq 300 \\ 1000 + 5(x - 300) & , 300 < x \leq 500 \\ 1000 + 5(200) + 10(x - 500) & 500 < x \leq 600 \\ 2000 + 10(x - 500) \\ 1000 + 1000 + 1000 + 15(x - 600) & x > 600 \\ 3000 + 15(x - 600) \end{cases}$$

5. (5 points) You have \$300,000 deposited in a bank paying 5% compounded monthly. How long will it take for your investment to grow to \$350,000?

$$A = P(1+i)^n$$

$$350000 = 300000 \times \left(1 + \frac{0.05}{12}\right)^{12t}$$

$$\log \frac{7}{6} = \log (1.00417)^{12t}$$

$$\log \left(\frac{7}{6}\right) = 12t \log 1.00417$$

$$t = \frac{\log \left(\frac{7}{6}\right)}{12 \times \log 1.00417} = 3.08$$

~ 3 years

20 years 6%

25 years

6. (10 points) A person wants to establish an annuity for retirement purposes. He wants to make quarterly deposits for 20 years so that he has \$1,000,000 in his account. He then plans to make quarterly withdrawals for 25 years reducing the balance to zero. The annuity earns 6% interest compounded quarterly? FV
PV

a. How much should he deposit each quarter for 20 years in order to accumulate the required amount?

$$\frac{0.06}{4} = 0.015 \quad PMT = 1000000 \times \left[\frac{0.015}{(1+0.015)^{4 \times 20} - 1} \right]$$

$$PMT = \$6548.323 \checkmark$$

$$6548.323 \times 4 \times 20$$

$$523865.84$$

$$= 476134.16$$

b. How much can he withdraw each quarter for 25 years in order to reduce the balance to zero?

$$PV = 1000000$$

$$PMT = 1000000 \times \left[\frac{0.015}{(1+0.015)^{4 \times 25} - 1} \right]$$

$$PMT = \$19370.57 \checkmark$$

$$19370.57 \times 4 \times 25$$

c. What is the total amount of interest earned during the 45-year period?

$$\begin{aligned} & \text{Total withdrawals} - \text{Total payments} \\ & = (19370.57 \times 4 \times 25) - (6548.323 \times 4 \times 20) \\ & = \$1913191.16 \checkmark \end{aligned}$$

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7. (7 points) In order to buy a car, Yassine takes a loan of \$35,000 to be repaid over 4 years by monthly payments of \$821.98. The interest rate charged by the bank is 6% compounded monthly. P1

a. After 12 payments are made, what is the unpaid balance of the loan? PMT

$$\frac{r}{n} = \frac{0.06}{12}$$

0.005

$$PV = 821.98 \times \left[\frac{1 - (1 + 0.005)^{-36}}{0.005} \right]$$

$$= \$ 27019.31$$

b. How much interest was paid during the first year?

$$(PMT \times m) - (FV - PV)$$

$$(821.98 \times 12) - \left[821.98 \times \left(\frac{1 - (1 + 0.005)^{-12}}{0.005} \right) - 821.98 \times \frac{1 - (1 + 0.005)^{-12}}{0.005} \right]$$

$$9863.76 - [2000.16 - 27019.31]$$

$$9863.76 - 1980.85 = \$ 7882.91$$