College of Arts and Sciences

Exam I– Spring 2008

Mathematics for Business I (MTH 101)
Section: 1-7

50
Excellent
(I made +5)

Instructor Name	Sec. Number	Class Time
Dr. T. Abualrub		
Dr. A. Badawi		
Dr. G. Gunatillake		11:00 – 11:50. am
Dr. G. Leduc		

Date: Wednesday March 26, 2008

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This examination has **5** pages plus the formula sheet and this cover page. Before you start the examination please verify the number of pages.

No Questions are allowed during the examination

Student signature: Walaa

1. (8 points) What amount will an account have after 6 years if \$9,000 is invested at an annual rate of 5%

(a) (4 points) Simple interest?

$A = P(1+rt)$ $\quad A = \$11,700$

$A = 9000(1+0.05 \times 6)$

(b) (4 points) Compounded quarterly?

$A = P(1+i)^n$ $\quad i = \frac{0.05}{4}$ $\quad n = mt = 4 \times 6 = 24$

$A = 9,000\left(1+\frac{0.05}{4}\right)^{24}$

$A = \$\ 12,126.159$

2. (7 points) If you can afford monthly deposits of \$500 into an account paying 6% compounded monthly, how long will it be until you will have \$30,000? Fv

51.6

pmt = \$500 $\quad i = \frac{0.06}{12}$ $\quad Fv = 30,000$ $\quad Fv = pmt\left(\frac{(1+i)^n - 1}{i}\right)$

t is unknown.

$30,000 = 500\left(\frac{\left(1+\frac{0.06}{12}\right)^{12t} - 1}{\frac{0.06}{12}}\right)$ $\quad n = 12t$, $t = n/12$.

$\frac{30,000}{500} \times \frac{0.06}{12} + 1 = \left(1+\frac{0.06}{12}\right)^{12t}$

$\frac{300}{5} \times \frac{0.06}{12} + 1 = 1.005^n$

$\log\left(\frac{300}{5} \times \frac{0.06}{12} + 1\right) = n \log 1.005$

$n = \frac{\log\left(\frac{300}{5} + \frac{0.06}{12} + 1\right)}{\log 1.005}$

$n = 52.6$

$t = \frac{52.6}{12}$ $\quad t \approx 4.3$ years

3. (4 points) A doughnut shop has a fixed cost of \$210 per day and a variable cost of \$0.20 per doughnut.

(a) (2 points) Find the total daily cost of producing x doughnuts.

cost $= mx + b$

$C(x) = 0.20x + 210.$

(b) (2 points) How many doughnuts can be produced for a total daily cost of \$400?

$0.20x + 210 = 400$

$0.20x = 190$

$x = \frac{190}{0.20}$

$x = 950$ doughnuts.

4. (14 points) The research department in a company that manufactures clock radios established the following price-demand, and cost functions:

$p(x) = 50 - 1.25x$ Price-demand function

$C(x) = 140 + 10x$ Cost function

where x is in thousands of units, and $C(x)$ is thousands of dollars. All functions have $1 \leq x \leq 30$.

(a) (2 points) Write the revenue function.

Revenue = price × quantity.

$R(x) = x(50 - 1.25x)$

$R(x) = 50x - 1.25x^2$

(b) (2 points) Find the maximum revenue.

max revenue = $\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)$

revenue function = $50x - 1.25x^2$ $\quad b = 50$, $a = -1.25$

number of items that will produce max revenue

$= \frac{-b}{2a} = \frac{-50}{-2.5}$

$= 20$ in thousand of units

max revenue = $50(20) - 1.25(20)^2$ $=$ max revenue 500 in thousands of dollars.

(20, 500)

(c) (3 points) Determine the break-even point(s).

profit = $R(x) - C(x)$

$P(x) = 50x - 1.25x^2 - (140 + 10x)$

$P(x) = 50x - 1.25x^2 - 140 - 10x$ = profit equation. $= 40x - 1.25x^2 - 140$

BEV: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = -1.25$ $b = 40$ $c = -140$

$\frac{-40 \pm \sqrt{40^2 - (4 \times -1.25 \times -140)}}{2(-1.25)}$

$\frac{-40 \pm 30}{-2.5}$ = $BEV_1 = 28$, $BEV_2 = 4$

BEP:
Substitute BEV in either cost or revenue function

$= (x) = 140 + 10x$

① $140 + 10(28) = 420$

② $140 + 10(4) = 180$

BEP = (28, 420) (4, 180)

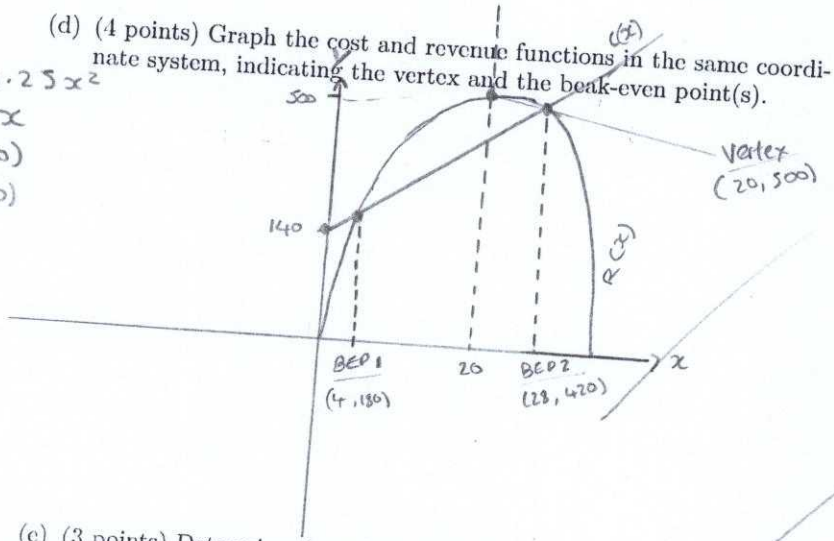
(d) (4 points) Graph the cost and revenue functions in the same coordinate system, indicating the vertex and the break-even point(s).

Revenue: $50x - 1.25x^2$

cost: $140 + 10x$

Vertex = (20, 500)

BEP = (28, 420) (4, 180)



(cost, when $x = 0$
$= 140$ (y intercept)

revenue curve facing downward because the a value is negative and we are finding the maximum revenue

(e) (3 points) Determine the values of x where a profit occurs.

The breakeven values of x are 4 thousands of units and 28 thousands of units. where Revenue = cost

Values of x where a profit occurs is between these two values

$4 < x < 28$ in thousand of units.

in thousands of units

5. (7 points) The table below shows the electricity rates charged by XYZ utilities in the summer month

$3.00 for the first 20 kWh or less
$0.0570 per kWh for the next 180 kWh
$0.0346 per kWh for all over 200 kWh

(a) (5 points) Write a piecewise definition for the monthly charge $S(x)$ (in dollars) for a customer who uses x kWh in a summer month.

$$\begin{cases} 3.00 & 0 \leq x \leq 20 \\ 0.0570x + 1.86 & 20 < x \leq 200 \\ 0.0346x + 6.34 & x > 200 \end{cases}$$

① $3.00 + 0.0570(x - 20)$.
$0.0570x + 1.86$

② $3.00 + 0.0570(180) + 0.0346(x - 200)$.
$0.0346x + 6.34$

(b) (2 points) How much is the monthly charge for a customer who uses 640 kWh?

we use the 3rd piecewise function, because 640 is above 200.

$0.0346(640) + 6.34 = \$28.484$ per month

6. (10 points) Suppose you borrow \$20,000 at 8% interest rate compounded annually, which is to be amortized over 5 years in equal yearly payments.

$pmt = pv\left(\frac{i}{1-(1+i)^{-n}}\right)$

(a) (3 points) Find the interest paid during the 5 years.

$(i) = \frac{0.08}{1}$ $t = 5$ $pv = \$20,000$

$n = mt \cdot 5$

$pmt = \frac{20,000 \times \frac{0.08}{1}}{1-(1+\frac{0.08}{1})^{-5}} = \$5009.12$ yearly payment is \$5009.12

* Interest paid:
money paid − money borrowed
(5 yearly payments)
$= (5009.12 \times 5) - \$20,000$
$\$25045.6 - \$20,000 = \$5,045.6$.

(b) (7 points) Find the interest paid during the third year.

Interest paid during third year

$(i) = \frac{0.08}{1}$

$pmt = 5009.12$

yearly payment − (pv after 2 years − pv after 3 years)

yearly payment → 5009.12

pv after 2 years: $t = 3$ → $5009.12\left(\frac{1-(1+\frac{0.08}{1})^{-3}}{\frac{0.08}{1}}\right) = 12908.9$

pv after 3 years: $t = 2$ → $5009.12\left(\frac{1-(1+\frac{0.08}{1})^{-2}}{\frac{0.08}{1}}\right) = 8932.5$

$Interest = 5009.12 - (12908.9 - 8932.5) = \$1032.72$