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Final Exam: MTH 530, Fall 2017

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QUESTION 1. Let D be a group with $5^2 \cdot 7^2$ elements. Is D a nilpotent group? explain. If yes, then up to isomorphism find all possible structures of D. (note that every group with p^2 elements is abelian where p is prime)

QUESTION 2. 1)Let D be an infinite simple group and H be a subgroup of D. Assume that $H \neq \{e\}$ and $H \neq D$. Prove that H has infinitely many distinct left cosets.

QUESTION 3. 2) Does A_6 have a subgroup, say H, with 90 elements? If yes, then if $a \in H$ and a is of maximal order, say m, then what is m? If no, then convince me.

3) Consider the group $(Z_p^*, .)$ where p is prime and $p \geq 5$. Let $F = \{x^2 \mid x \in D\}$. Prove that F is a subgroup of Z_p^*. Find $[D : F]$. If $p - 1 \notin F$, prove that $a \in F$ or $p - a \in F$.

4) Prove that every group of order 72 is not simple.

QUESTION 4. 1)Let $(D, *)$ be a simple group. Assume that H_1, H_2 are subgroups of D where $[D : H_1] = p_1$ and $[D : H_2] = p_2$ for some prime integers p_1, p_2. Prove that $p_1 = p_2$.

2) By Krull-Schmidt Theorem, $F = U(3^2.5^2.11^2)$ is isomorphic to a product of irreducible normal subgroups and this product is unique (up to isomorphism) (i.e., write F as a product of its invariant factors).

3) Let F be an infinite finitely generated abelian group where 4 is the rank of the free-torsion part of F. Assume that if $a \in F$ and a is of finite order, then $|a| = 3$ or 9 or 1. If F has exactly 18 elements of order 9 and exactly 8 elements of order 3, then up to isomorphism, determine the structure of F.

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