Problem One — 18 Points

Find each of the following derivatives **(do not simplify)**:

(a) $y = \left(\sqrt{x} + 4x^2\right)\left(x - \frac{1}{x}\right)$

(b) $f(x) = \sqrt{(x^2+3)(x-4)}$

(c) $f(x) = \left(\frac{8x^2 - 4}{1 - 9x^3}\right)^5$

Problem Two — 15 Points

Evaluate the following limits:

(a) $\lim_{x \to 0} \frac{\sqrt{2-x} - \sqrt{2}}{x}$

(b) $\lim_{x \to \frac{3}{2}^+} \frac{|2x-3|}{3-2x}$

(c) $\lim_{x \to -2} \frac{3x^2 - x - 10}{x^2 - x - 2}$

Problem Three — 12 Points

(a) Find the equation of the tangent line to the graph of $f(x) = 2x - \frac{4}{\sqrt{x}}$ at the point (4,6).

(b) At what point(s) does the function $f(x) = (x^2 - 4)^3$ has a horizontal tangent line?

Problem Four — 10 Points

Use the definition of the derivative to find the derivative of $f(x) = x^2 - 1$

Problem Five — 20 Points

Suppose that the price-demand equation and the total cost (in dollars) for manufacturing x

TV sets are given respectively by :

$$x = 6000 - 30p \qquad \text{and} \quad C(x) = 72000 + 60x$$

(a) Express the price p in terms of x and find the domain of p.

(b) Find the total revenue in terms of x.

(c) Find the profit function in terms of x.

(d) Evaluate the marginal profit at $x = 1500$ and $x = 3000$ and interpret the results?

(e) Find the exact profit from the sale of the 801^{st} TV set.

(f) Use the marginal profit function to approximate the profit from the sale of the 801^{st} TV set.

Problem Six — 9Points

For the function $f(x) = \dfrac{x+1}{x^2 - 2x - 3}$, Determine the following:

(a) The domain of $f(x)$.

(b) The vertical Asymptotes of $f(x)$.

(c) The horizontal Asymptotes of $f(x)$.

Problem Seven — 20pts

For the function $f(x) = -\frac{1}{4}x^4 + x^3$, find the following:

(a) Domain

(b) x and y intercepts.

(c) Intervals where f is increasing and decreasing and find the local maxima and minima if any.

(d) Intervals where f is concave upward and concave upward and downward and find any inflection points.

(d) Graph $f(x)$.