I wish that I can write as you do

Excellent +++

100/100

TEST NUMBER ONE FOR MTH102 SPRING007

AYMAN BADAWI

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QUESTION 1. 20 points

(1) *Find* $\lim_{x\to 3^+}\frac{|x-3|}{x-3} = \frac{0}{0}$

$$\begin{cases} x-3 & x\geq 3 \\ -(x-3) & x<3 \end{cases}$$

$$= \lim_{x\to 3^+}\frac{x-3}{x-3} = \lim_{x\to 3^+} 1 = \boxed{1}$$

(2) *Find* $\lim_{h\to 0}\frac{\sqrt{3+h}-\sqrt{3}}{h} = \frac{0}{0}$

$$= \lim_{h\to 0}\frac{\sqrt{3+h}-\sqrt{3}}{h}\cdot\frac{\sqrt{3+h}+\sqrt{3}}{\sqrt{3+h}+\sqrt{3}} = \lim_{h\to 0}\frac{3+h-3}{h(\sqrt{3+h}+\sqrt{3})}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{3+h}+\sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{3}} = \boxed{\frac{1}{2\sqrt{3}}}$$

(3) *Find* $\lim_{x\to -3}\frac{x^2-9}{x^2+5x+6} = \frac{0}{0}$

$$= \lim_{x\to -3}\frac{(x-3)(x+3)}{(x+2)(x+3)} = \frac{-3-3}{-3+2} = \frac{-6}{-1} = \boxed{6}$$

(4) *Find*

$$\lim_{x\to -1}\frac{x+4}{x+2} = \frac{-1+4}{-1+2} = \frac{3}{1} = \boxed{3}$$

$-\frac{1}{4}+\frac{28}{4}=\frac{27}{4}$

QUESTION 2. (10 points) *Find the equation of the tangent line to the curve of* $f(x)=\sqrt{x+3}+5$ *at* $x=1$.

$f(x)=\sqrt{x+3}+5 \qquad y=f(1)=\sqrt{4}+5=2+5=7$

$m=f'(x)=\frac{1}{2\sqrt{x+3}} \qquad (1,7)$

$m=f'(1)=\frac{1}{2\sqrt{4}}=\frac{1}{2\cdot 2}=\boxed{\frac{1}{4}}$

$y-7=\frac{1}{4}(x-1)$

$y=\frac{1}{4}x-\frac{1}{4}+7$

$\boxed{y=\frac{1}{4}x+\frac{27}{4}}$

$g=\sqrt{3x+6}$

$g'=\frac{3}{2\sqrt{3x+6}}$

QUESTION 3. (14 points) *Find the first derivative of* $f(x)$ *(DO NOT SIMPLIFY)*

1) $f(x)=\underbrace{(\sqrt{3x+6})^5}_{u}\underbrace{(x^2-6x+12)}_{v}$

$u=(\sqrt{3x+6})^5 \qquad v=x^2-6x+12$

$u'=\frac{3(5)(\sqrt{3x+6})^4}{2\sqrt{3x+6}} \qquad v'=2x-6$

$f'(x)=u'v+v'u$

$\boxed{f'(x)=\frac{15(\sqrt{3x+6})^4}{2\sqrt{3x+6}}(x^2-6x+12)+(2x-6)(\sqrt{3x+6})^5}$

$u'=\frac{15(\sqrt{3x+6})^4}{2\sqrt{3x+6}}$

2) $f(x)=\frac{x+7}{6x-7}=\frac{u}{v}$

$u=x+7 \qquad v=6x-7$

$u'=1 \qquad v'=6$

$f'(x)=\frac{u'v-v'u}{v^2}$

$\boxed{f'(x)=\frac{6x-7-6(x+7)}{(6x-7)^2}}$

$f'(x)=\frac{6x-6x-49}{(6x-7)^2}=\frac{-49}{(6x-7)^2}$

QUESTION 4. (20 points) *Let* $f(x) = x^4 - 4x^3 + 2$ *on the interval* $[-1, 4]$

(1) *Where does* $f(x)$ *increase on the given interval? Where does* $f(x)$ *decrease on the given interval?*

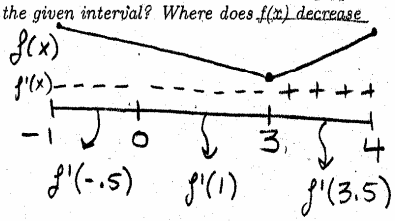
$f'(x) = 4x^3 - 12x^2$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0 \qquad x = 3$

both included in the interval



$f(x)$ increases on $x = [3, 4]$

$f(x)$ decreases on x $[-1, 3]$

(2) *Find* Local Max. *and* Local Min. *values of* $f(x)$ *on the given interval.*

x	-1	0	3	4
f(x)	7	2	-25	2

$f(x)$ has a local max at $x = -1, x = 4$

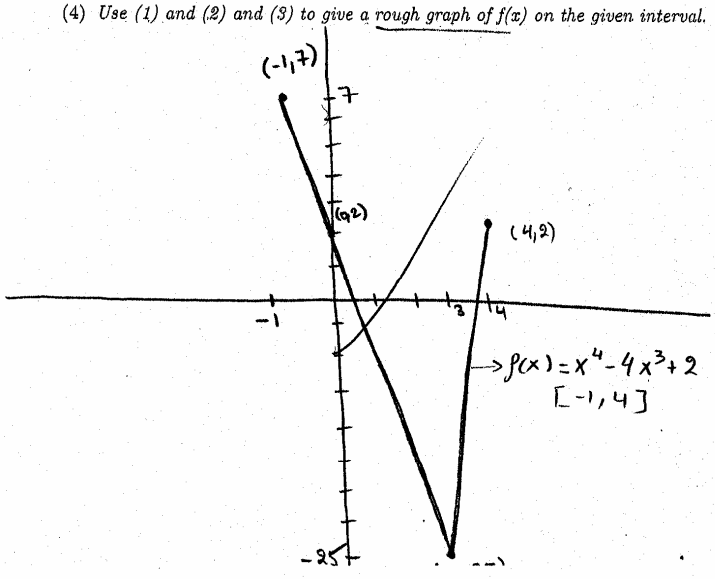
$f(x)$ has a local min at $x = 3$

(3) *Find the absolute* max *and the absolute min of* $f(x)$ *on the given interval.*

$f(x)$ has absolute max at $x = -1$

$f(x)$ has absolute min at $x = 3$

(4) *Use (1) and (2) and (3) to give a rough graph of* $f(x)$ *on the given interval.* $f''(x) =$



QUESTION 5. (16 points) *let* $f(x) = \frac{3x^2+3}{-2x^2+8}$.

1) Find the Horizontal asymptote of $f(x)$

$f(x)$ has a H.A. at $y = \frac{3}{-2} = -\frac{3}{2} = -1.5$ $\boxed{y=-1.5}$

2) Find the Vertical asymptotes of $f(x)$.

$f(x)$ has a V.A. at $\boxed{x=2 \& x=-2}$

$-2x^2+8=0$

$\frac{-2x^2}{-2} = \frac{8}{2}$

$x^2 = 4$

$x = \pm 2 \Rightarrow x=2, \; x=-2$

numerator $\neq 0$

$x=2$	$x=-2$
$3(2)^2+3$	$3(-2)^2+3$
$=15\neq 0$	$=15\neq 0$

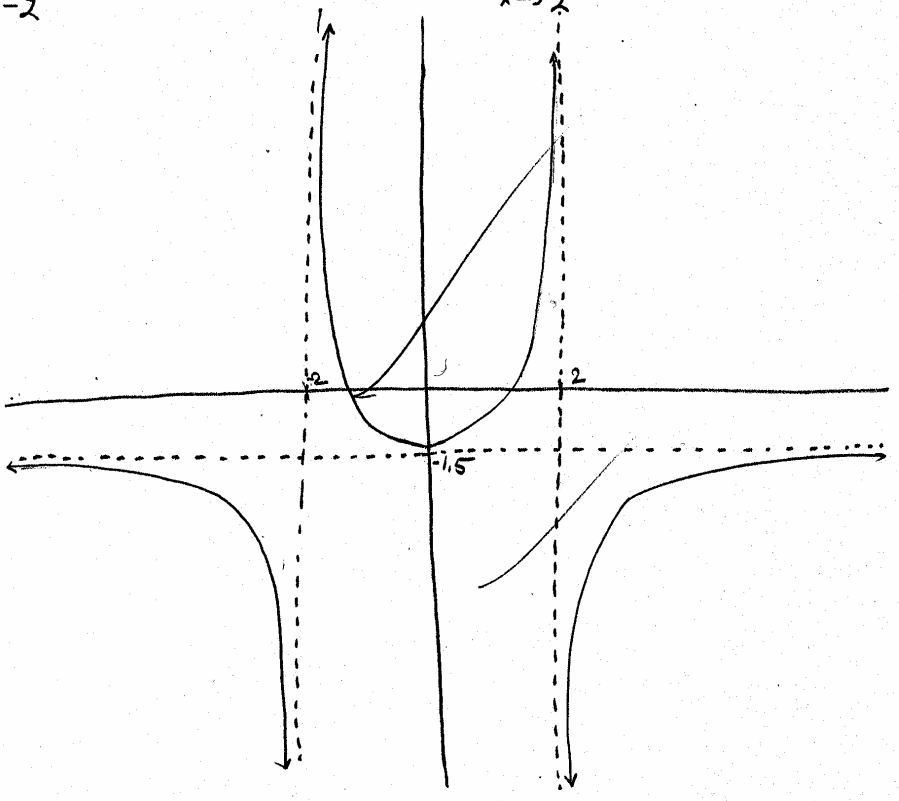
3) Sketch $f(x)$.

$\lim_{x\to -2^+} \frac{3x^2+3}{-2x^2+8} = \frac{+}{+} = +\infty$

$\lim_{x\to -2^-} \frac{3x^2+3}{-2x^2+8} = \frac{+}{-} = -\infty$

$\lim_{x\to 2^+} \frac{3x^2+3}{-2x^2+8} = \frac{+}{-} = -\infty$

$\lim_{x\to 2^-} \frac{3x^2+3}{-2x^2+8} = \frac{+}{-} = +\infty$



QUESTION 6. *(20 points) Let* $P(x) = x^3 - x^2 + 100$ *be the profit function on selling* x *units.*

(1) *What is the marginal profit function? What is the profit when* $x = 10$*?* $P'(x)$? $P(10)$?

$P'(x) = 3x^2 - 2x$ $\quad P(10) = (10)^3 - (10)^2 + 100 = \$1,000$

(2) *Find the marginal profit value when* $x = 10$. $P'(10)$?

$P'(10) = 3(10)^2 - 2(10) = \$280$

(3) *Use (1) and (2) to approximate the total profit when* $x = 11$.

$P(11) = P(10) + P'(10) = 1,000 + 280 = \$1,280$

(4) *Find the Marginal Average Profit Function. Then find the Marginal Average Profit when* $x = 10$. $\overline{P}'(x)$? $\overline{P}'(10)$? $100x^{-1} = -100x^{-2}$

$$\overline{P}(x) = \frac{x^3 - x^2 + 100}{x} = x^2 - x + \frac{100}{x}$$

$$\left(\overline{P}(x)\right)' = 2x - 1 - 100x^{-2} = 2x - 1 - \frac{100}{x^2} = 2x - \frac{100}{x^2} - 1$$

$$\left(\overline{P}(10)\right)' = 2(10) - \frac{100}{10^2} - 1 = 20 - 2 = 18$$

(5) *Use Part (4) above only to approximate the total profit when* $x = 11$. $P(11)$?

$\overline{P}(11) = \overline{P}(10) + \overline{P}'(10) = 100 + 18 = 118$ $\quad \overline{P}(10) = (10)^2 - (10) + \frac{100}{10} = 100$

$P(11) = 118(11) = \$1,298$

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$\overline{P}(11) \neq P(11)$ $\quad P(11) = 118(11) = \$1298$

$\frac{\overline{P}(11)}{x} = P(11)$

$\overline{P}(11) = xP(11)$