1. (15 Points) Find a particular solution using the method of undetermined coefficients.

$$y'' + 4y = 6\sin 2x + 3x$$

2. (15 Points) Set up the appropriate form of the particular solution y_p, but do not determine the values of the coefficients.

a. $y''' - 2y'' + 2y' = 5x^2 + e^x + \cos 3x$

b. $y^{(4)} - y'' = 4x + 2xe^{-x}$

3. (15 Points) Show that $y_1 = x$ is a solution of the differential equation

$$\left(x^2 + 1\right) y'' - 2xy' + 2y = 0$$

Use reduction of order to find a second linearly independent solution of the differential equation.

4. (15 Points) Find the general solution of the given differential equation.

$$x^2 y'' + 9xy' - 20y = \frac{5}{x^3}$$

5. (15 Points) A mass weighing 4 lb stretches a spring 2 ft. The mass is attached to a viscous damper with a damping force numerically equals to the instantaneous velocity. If the mass is set in motion 1 ft above the equilibrium position with a downward velocity of 8 ft/sec.

a. Find the equation of motion.

b. Is the system critically damped, under-damped or over-damped?

c. Determine when the mass first returns to its equilibrium position.

6. (15 Points) Use the power series method to solve the given initial-value problem. Determine the recurrence relation and the first six terms.

$$(1+x^2)\,y'' + y = 0, \qquad y(0) = 1,\;\; y'(0) = 0$$

7. (10 points)

A. For what parameters a and b will the differential equation

$$y'' + ay' + by = 0$$

have the function $y = 3e^{-7x}\cos(10x)$ as a solution?

B. Assume that y_1 and y_2 are solutions of the differential equation

$$y'' + p(x)y' + q(x)y = 0$$

where $p(x) \neq 0$ and $q(x) \neq 0$. Prove that if y_1 and y_2 have a common point of inflection a in some interval I, then the two solutions are linearly dependent.