Name——————————————, ID ————————

Exam TWO, MTH 205, SPRING 2009

Ayman Badawi

QUESTION 1. (10 points) Solve: $(x+1)y' + (x+2)y = e^{4x}$

QUESTION 2. (10 points) Solve : $y' = e^{x+y}$

QUESTION 3. (15 points) Solve : $xy' - y = y(1 + \sqrt{y})$

QUESTION 4. (10 points) Solve: $\dfrac{dy}{dx} = \dfrac{ysin(x)+3y^2-x^2}{cos(x)-6yx+y^3}$

QUESTION 5. (20 points) Solve : $y^{(2)} - 4y = \frac{-8}{1+e^{2x}}$

QUESTION 6. (15 points) A tank has a capacity of 600 liters, it contains 200 liters in which 30 grams of salt is dissolved. A mixture containing 2 grams of salt per liter is pumped into the tank at rate of 4 liter/min; the well-mixed solution is pumped out at rate 2 liter/min. Find the amount of salt, $A(t)$, in the tank at any time t. Find the concentration of salt in the tank at t = 2. When does an overflow occur?

QUESTION 7. (20 points) An object weighing 64 pounds stretches a spring 2 feet. The mass is initially released from rest in the equilibrium position. The surrounding medium offers a force that is numerically equal 16 times the instant velocity. Find the equation of motion, $x(t)$, at any time time if the object is driven by a constant external force equal to $f(t) = 20$.

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Exam ONE, MTH 205, SPRING 2009

Ayman Badawi

QUESTION 1. (**25 points**) 1)Find $\ell\{2^{3x+2}\}$

2) Find $\ell\{x^2U(x-2)\}$

3) USE convolution to find $\ell^{-1}\{\frac{1}{s^2-s}\}$

4)Find $\ell^{-1}\left\{\frac{3e^{-2s}}{(s-4)^2+9}\right\}$

5) Find $\ell^{-1}\left\{\frac{s+5}{(2s+6)^3}\right\}$

QUESTION 2. (**10 points**) Solve $y^{(2)} + 9y = 3, y(0) = y'(0) = 0$

QUESTION 3. (**15 points**) Solve for y(x), $y'(x) = e^{2x} - \int_0^x e^{2x-2r} y(r)\, dr, y(0) = 0$

QUESTION 4. (**15 points**) Solve for $x(t)$ and $y(t)$ if $x'(t) - 0.5y(t) = t$ and $x(t) + \int_0^t y(r)\, dr = 2t^2, x(0) = y(0) = 0$

QUESTION 5. (**15 points**) Find the general solution to $y^{(3)} + 2y^{(2)} + y' = 0$, USE THE HOMOGENEOUS METHOD.

QUESTION 6. (**20 points**) a) Find the general solution to $y^{(2)} + 16y = 0$

b) If $y(0) = 0$ and $y'(\pi/8) = 0$, what will be the solution of the D.E in part (a)? Does that contradict one of the Theorem in the book? EXPLAIN (Note $sin(\pi/2) = 1, cos(\pi/2) = 0, sin(0) = 0, cos(0) = 1$)

c) If $y(0) = 0$ and $y'(\pi/8) = 1$, what will be the solution of the D.E. in part (a)? Does that contradict one of the Theorem in the book? EXPLAIN

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Q1. Solve the following Differential Equations: **(14 marks)**

(a) $$y' = xe^{x+2y}$$

(b) $$4x^2 y' - 6xy = 2yx(1 + 8e^{2x}\sqrt{y})$$

(c) $xy'' - 2y' = 0$ **(5 marks)**

(8 marks)

Q2. Write the form of the particular solution you would choose in the method of undetermined coefficients for the following:

(a) $y' + y = e^{x}$

(b) $y'' + y' + y = x\cos x$

(8 marks)

Q3. Solve the following differential equation by Variation of parameters:

$$x^2 y'' - 3xy' + 3y = x^4 \ln(x)$$

(10 marks)

Q4. Use Newton's law of cooling/warming to solve the following problem.

a) Ahmed put an elephant inside a refrigerator. The temperature inside the refrigerator is 5^0 F. After 1 minute the elephant cools down to 55^0 F and after 5 minutes it cools down to $30^{\ 0}$ F. What was the temperature of the elephant before it was put inside the refrigerator?

b) To put a giraffe inside the refrigerator Ahmed took the elephant from the refrigerator and placed it outside 9 minutes after it was first put inside. What is the elephant's temperature at this time?

Q5. (a) Use laplace transform to solve the following initial value problem **(12 marks)**

$$y'' - 2y' + y = e^t \qquad y(0) = 0, y'(0) = 5$$

(b) $y' + y = f(t)$, where $f(t) = \begin{cases} 0 & 0 \le t < 1 \\ 5 & t \ge 1 \end{cases}$,

$$y(0) = 0$$

Q6. Find the following laplace transforms/inverse laplace transform. **(15 marks)**

(a) $L\{te^{-t}\sin t\}$

(b) $L^{-1}\left\{\dfrac{s+\pi}{s^2+\pi^2}e^{-s}\right\}$

(c) $L\{t\int_0^t \sin zdz\}$

Q7 (a) Rewrite using unit step functions : **(10 marks)**

$$f(t)=\begin{cases} 0 & 0\le t\le 2\pi \\ \cos(t) & 2\pi\le t\le 5\pi/2 \\ 2-\sin(3t) & t>5\pi/2 \end{cases}$$

(b) Use the **convolution theorem** to find $L^{-1}\left\{\frac{1}{s(s-a)^2}\right\}$

Hint : $L^{-1}\left\{\frac{1}{(s-a)^2}\right\}=te^{at}$

Q8. Use Laplace to solve for $x(t)$ and $y(t)$ such that **(8 marks)**

$$x' + y(t) = 0 \qquad x(0) = 0$$

$$x(t) - y'(t) = -1 \qquad y(0) = 0$$

(6 marks)

Q9. An object weighing 16 pounds stretches a spring 2 feet. The mass is initially released from rest one foot below the equilibrium position. The surrounding medium offers a force that is numerically equal 4 times the instant velocity. Find the equation of motion $x(t)$ at any time t.

Q10. Solve the given integral equation for $y(t)$: **(6marks)**

$$y(t) + \int_0^t (t-z)^2 y(z)dz = t^3 + 3$$

Formula Sheet

1) $L\{1\} = \frac{1}{s}$ $\quad$ $L\{t^n\} = \frac{n!}{s^{n+1}}$ **, n is a positive integer**

2) $L\{\sin kt\} = \frac{k}{s^2 + k^2}$ $\quad$ $L\{\cos kt\} = \frac{s}{s^2 + k^2}$

3) $L\{e^{at}\} = \frac{1}{s-a}$

4) $L\{e^{at} f(t)\} = F(s)\big|_{s \to s-a}$ $\quad$ $L^{-1}\left\{F(s)\big|_{s \to s-a}\right\} = e^{at} f(t)$

5) $L\{g(t)U(t-a)\} = e^{-as} L\{g(t+a)\}$

$L^{-1}\{e^{-as} F(s)\} = f(t-a)U(t-a)$

6) $L\{U(t-a)\} = \frac{e^{-as}}{s}$ $\quad$ $L^{-1}\{\frac{e^{-as}}{s}\} = U(t-a)$

7) $L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - .... - f^{(n-1)}(0)$

8) $L\{t^n f(t)\}(s) = (-1)^n \frac{d^n F(s)}{ds^n}$ $\quad$ $L^{-1}\{\frac{d^n F(s)}{ds^n}\} = (-1)^n t^n f(t)$

9) $L\{f(t) * g(t)\} = F(s).G(s)$ $\quad$ $f(t) * g(t) = \int_0^t f(\tau) g(t-\tau) d\tau$

10) $L\{\int_0^t f(\tau) d\tau\} = \frac{F(s)}{s}$ $\quad$ $L^{-1}\{F(s).G(s)\} = f(t) * g(t)$

11) $L\{\delta(t)\} = 1$ $\quad$ $L\{\delta(t-a)\} = e^{-as}$

12) **If** $f(t)$ **is periodic with period** T **then**

$$L\{f(t)\} = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t) dt$$

13) $L\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$

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Review for Exam TWO, MTH 205, SPRING 2009

Ayman Badawi

QUESTION 1. A body at a temperature of 50 F is placed outdoors where the temperature is 100 F. If after 5 minutes the temperature of the body is 60 F, use the Newton's law of cooling to find an expression for the temperature T of the body after t minutes.

QUESTION 2. A large tank is filled with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at 10 gal/min.

(a) Find the number of pounds of the salt in the tank at time t. (b) What is the concentration of the solution in the tank at t = 5 min? (c) When is the tank empty?

QUESTION 3. Initially 100 milligrams of radioactive substance were present. After 6 hours the mass had decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours.

QUESTION 4. Classify each of the following differential equations as, separable, exact, linear, homogeneous, or Bernoulli and solve it. (a) $2xyy' + y^2 = 2x^2$ (b) $y' = xy + \sqrt{y}$ (c)$y' = \frac{5x+4y}{8y^3-4x}$

(d) $(1+x)y' + 2y = 3x + 3$ (e) $y' = \sqrt{y+x}$ (f)$y' = \frac{x^2+2}{y}$

(g) $y' = \frac{1}{x(x-y)}$(h) $xy' = 6y + 12x^4y^{2/3}$ (i)$y' = \frac{2+ye^{xy}}{2y-xe^{xy}}$

QUESTION 5. Solve: $y^{(2)} + 2y' + 2y = 20cos(2t)$, $y(0) = y'(0) = 0$.

QUESTION 6. Solve : $y^{(2)} + 4y = 2sin(2t)$, $y(0) = 1, y'(0) = 0$.

QUESTION 7. Solve: $y^{(2)} + 1 = tan(x)$.

QUESTION 8. A mass weighing 64 pounds stretches a spring 0.32 ft. The mass is initially released from a point 8 inches above the equilibrium position with downward velocity of 5 ft/sec.

(a) Find the equation of the motion. (b) Find the amplitude, natural frequency, and period? (c) At what time does the mass pass through the equilibrium position heading downward for the second time? (d) At what times the mass attain its extreme displacements above the equilibrium position?

QUESTION 9. A 16 - pound weight stretches a spring 2 feet. Initially the weight starts from rest 2 feet below the equilibrium position. Determine the differential equation and the initial conditions of the motion, if the surrounding medium offers a damping force numerically equal to the instantaneous velocity and the weight is driven by an external force equal to $f(t) = 5cos(2t)$.

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Review for Exam ONE, MTH 205, SPRING 2009

Ayman Badawi

QUESTION 1. (i) Find $\ell\{5^{2x+8}\}$

(ii) Find $\ell\{xe^{2x}sin(3x) + 3^{-x}cos(4x)\}$

(iii) Find $\ell\{U(x - \pi/2)e^{x+\pi}sin(x)\}$

(iv) Find $\ell^{-1}\{\frac{se^{-6s}}{s^2+9}\}$

(v) Find $\ell^{-1}\{\frac{3s}{(s+3)^2+16}\}$

(vi) Find $\ell^{-1}\{\frac{2}{s^2+6s+13}\}$

(vii) Find $\ell^{-1}\{\frac{1}{(3s+7)^2}\}$

(viii) Find $\ell^{-1}\{\frac{3s}{(5-2s)^3}\}$

(x) Find $\ell^{-1}\{\frac{e^{-3s}(2s-5)}{5s^2-20}\}$

QUESTION 2. Find the largest interval so that $\frac{x-9}{x^2-16}y^{(2)} + \sqrt{12-x}y' + xy = 10, y'(5) = 7, y(5) = -6$ has a unique solution.

QUESTION 3. 1)Use Laplace to solve $y^{(2)} + 2y' + y = e^x,\ y'(0) = y(0) = 0$

QUESTION 4. 1. Use LAPLACE to find the general solution to $y^{(2)} - 3y' + 2y = U(x-2)$

QUESTION 5. 1. Use Laplace to solve $y^{(2)} + 16y = 2sin4x, y'(0) = -0.5, y(0) = 0$ [Use the fact $\ell^{-1}\{\frac{2b^3}{(s^2+b^2)^2}\} = sinbx - bxcosbx$]

QUESTION 6. 1. Use laplace to find the general solution to $y^{(2)} + 6y' = 6 + 12e^x$.

QUESTION 7. Use LAPLACE to solve $2y^{(2)} + 6y' - 8y = 24e^{-x}, y(0) = 0, y'(1) = 0,$

QUESTION 8. Solve for $f(x)$ such that $f(x) = e^x + \int_0^x sin(t)f(x-t)\,dt$

QUESTION 9. Use Convolution to find $\ell^{-1}\{\frac{1}{s^2+5s+4}\}$

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<u>MTH205:</u> Review Problems / Final / Spring 09

Q1. Does the differential equation: $y' = y/x + \sqrt{x}$ possess a unique solution through the point $(0,0)$? Give reasons.

Q2. Find the critical points and phase portrait of $y' = y^2 - y^3$ and classify each point.

Q3. Classify each of the following differential equations as, separable, exact, linear, homogeneous, or Bernoulli. Then find the general solution of each of them.

a. $2xyy' + y^2 = 2x^2$, **b.** $y' = xy + \sqrt{y}$, **c.** $y' = \dfrac{x - 2xy}{3y^2 + x^2}$, **d.** $y' = e^{3x - 2y}$

Q4. Suppose that a large mixing tank initially holds 500 gallons of water in which 50 pounds of salt have been dissolved. Water is pumped into the tank at a rate of r_1 gal/min, and then when the solution is well stirred it is pumped out at a rate of r_2 gal/min. Determine a differential equation then solve it, for the amount $x(t)$ of the salt in the tank at any time t for each of the following cases:

a. $r_1 = r_2 = 4$, and the entering water is pure.
b. $r_1 = 3$, $r_2 = 2$, and the entering water contains salt with concentration 2 lb/gal.

Q5. Initially 100 milligrams of radioactive substance were present. After 6 hours the mass had decreased by 3%. If the rate of decay is proportional to the amount of the substance present at any time, find the amount remaining after 24 hours.

Q6 Find an interval around $x = 0$ for which the initial value problem

$$\sqrt{x+1}\; y'' + \frac{1}{4 - x^2} y' + y = \sin(x) \quad , \quad y(0) = 1 \quad , \quad y'(0) = 0$$

has a unique solution.

Q7. Find the general solution.

a. $x^3 y''' + 6x^2 y'' + 4xy' - 4y = 0$, **b.** $y^{(4)} + y'' = 0$,
c. $x^2 y'' - 4xy' + 6y = x^3$

Q8. Given that $y_1 = e^x$ is a solution of the differential equation.

$$(x+1)y'' - (x+2)y' + y = 0$$

Find the general solution.

Q9. Set up the appropriate form of the particular solution y_p, but do not determine the values of the coefficients.

$$y^{(4)} + 9y'' = 2x + (x^2 + 1)\sin(3x)$$

Q10. Use the method of undetermined coefficients to find the solution of the I.V.P.

$$y'' + 4y = 2\sin(2t) \quad , \quad y(0) = 1, y'(0) = 0$$

Q11. Use the variation of parameters method to find the general solution of the differential equation:

$$x^2 y'' - 3xy' + 4y = x^2 \ln x$$

Q12. A 32 – pound weight stretches a spring 32/5 feet. Initially the weight is released 1 foot below the equilibrium position with a downward velocity of 5 ft/sec. Determine the equation of the motion, if the surrounding medium offer a damping force numerically equal to 2 times the instantaneous velocity and the weight is driven by an external force equal to $f(t) = 12\cos 2t + 3\sin 2t$. Graph the transient and the steady-state solutions on the same coordinate axes.

Q13. A 64 – pound weight stretches a spring 0.32 foot. Initially the weight is released 2/3 foot above the equilibrium position with a downward velocity of 5 ft/sec.

- **a.** Determine the equation of the motion.
- **b.** What are the amplitude and the period of the motion?
- **c.** At what time does the mass pass through the equilibrium position for the second time?
- **d.** At what time does the mass attain its extreme displacement for the second time?

Q14. Determine a lower bound for the radius of convergence of the series solutions of the differential equation $(x+1)(x^2+2)y'' - xy' + 2y = 0$ about $x = 0$.

Q15. Find the first six terms of a series solution about $x = 0$ for the IVP

$$(x^2 + 2)y'' - y = 0, \quad y(0) = 2, \quad y'(0) = 3$$

Q16. Use translation and other theorems to find:

a) $L\{e^{at}\sin t\}$ b) $L^{-1}\left\{\dfrac{s}{(s+1)^3}\right\}$ c) $L\{g(t)\}, \; g(t) = \begin{cases} 0 \;, & 0 \le t < 1 \\ t^2 \;, & t \ge 1 \end{cases}$

d) $L^{-1}\{\dfrac{e^{-2s}}{s^3}\}$ e) $L\{t^3\cos 5t\}$

Q17. Use the Laplace transform to solve the following initial value problems

a. $y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 1$, where $f(t) = \begin{cases} 0 \;, & 0 \le t < \pi \\ 1, & \pi \le t < 2\pi \\ 0 \;, & t \ge 2\pi \end{cases}$

b. $y'' - 4y = e^t \quad , \quad y(0) = 1, y'(0) = 0$

c. $y'' + y = \delta(t - \pi), \; y(0) = 1, y'(0) = 0$

Q18. Use the Convolution Theorem to

a. evaluate $L\{\ t^2 * te^t\ \}$ and $L^{-1}\left\{\dfrac{1}{s^3(s-1)}\right\}$

b. solve $y' = 1 - \sin t - \int_0^t y(\tau)\ d\tau\,,\quad y(0) = 0$

Q19. Solve $y'' + 2y + 10y = f(t)\,,\ y(0) = 0,\quad y'(0) = 0,$ where f is periodic with period $T = 2\pi$ and $f(t) = \begin{cases} 1 & ,\ 0 \le t < \pi \\ -1 & ,\ \pi \le t < 2\pi \end{cases}$

Q20. Use Laplace Transform to solve the system

$$x' = 2y + e^t\,,\ \ x(0) = 1$$
$$y' = 8x - t,\ \ y(0) = 1$$

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Exam ONE, MTH 205, SPRING 2009

Ayman Badawi

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QUESTION 1. (25 points) 1)Find $\ell\{2^{3x+2}\}$

$$\Rightarrow \ell\{e^{(3x+2)\ln 2}\} = \{e^{3x\ln 2 + 2\ln 2}\} = \{e^{3x\ln 2} \cdot e^{2\ln 2}\}$$

$$= e^{2\ln 2}\,\ell\{e^{3\ln 2 x}\} = \boxed{\frac{e^{2\ln 2}}{s - 3\ln 2}} = 4\frac{1}{s-3\ln(2)}$$

2) Find $\ell\{x^2U(x-2)\}$

$(a=2)$

e^{-2s} $\Rightarrow g(x+2) = (x+2)^2$ ✓ ✓

$$\Rightarrow \ell\{g(x+2)\} = \ell\{(x+2)^2\} = \ell\{x^2+4x+4\} = \frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}$$

$$\Rightarrow e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)$$ ✓

3) USE convolution to find $\ell^{-1}\{\frac{1}{s^2-s}\}$

$$= \ell^{-1}\left\{\frac{1}{s(s-1)}\right\} = \left\{\frac{1}{s} \cdot \frac{1}{s-1}\right\}$$

$\frac{1}{s} \downarrow F(s)$, $\frac{1}{s-1} \downarrow G(s)$

$$= 1 * e^x$$

$$= \int_0^x e^r\,dr$$ ✓

$$\Rightarrow e^r\Big|_{r=0}^{r=x} = e^x - e^0 = \boxed{e^x - 1}$$

4)Find $\ell^{-1}\left\{\frac{3e^{-2s}}{(s-4)^2+9}\right\}$, $f(x-2)\,u(x-2)$

$\Rightarrow \ell^{-1}\left\{e^{-2s}\cdot\underbrace{\frac{3}{(s-4)^2+9}}_{F(s)}\right\}$ $\Rightarrow u(x-2)\sin(3x-6)\,e^{4x-8}$

$\Rightarrow f(x) = \sin 3x\, e^{4x}$

$\Rightarrow f(x-2) = \sin(3x-6)\, e^{4x-8}$

5) Find $\ell^{-1}\left\{\frac{s+5}{(2s+6)^3}\right\}$

$= \ell^{-1}\left\{\frac{s+5}{[2(s+3)]^3}\right\} = {}^{-1}\left\{\frac{s+5}{8(s+3)^3}\right\} = \frac{1}{8}\ell^{-1}\left\{\frac{s+5-2+2}{(s+3)^3}\right\} = \frac{1}{8}\ell^{-1}\left(\frac{1}{(s+3)^2}+\frac{2}{(s+3}\right.$

$= \frac{1}{8}\left(xe^{-3x} + x^2e^{-3x}\right) = \frac{xe^{-3x}}{8}(1+x)$

QUESTION 2. (10 points) Solve $y^{(2)} + 9y = 3, y(0) = y'(0) = 0$

Apply laplace

$\Rightarrow s^2Y_{(s)} - sy(0) - y'(0) + 9Y_{(s)} = \frac{3}{s}$ (with $y(0)\to 0$, $y'(0)\to 0$)

$\Rightarrow Y_{(s)}(s^2+9) = \frac{3}{s}$

$\Rightarrow Y_{(s)} = \frac{3}{s(s^2+9)}$

$\Rightarrow y_{(x)} = \ell^{-1}\left\{\frac{3}{s(s^2+9)}\right\}$

$= \ell^{-1}\left\{\frac{1}{s}\cdot\frac{3}{s^2+9}\right\}$

$= 1 * \sin 3x$

$\Rightarrow 1 * \sin 3x = \int_0^x \sin 3r\, dr$

$= -\frac{1}{3}\cos 3r \Big|_{r=0}^{r=x}$

$= -\frac{1}{3}(\cos 3x - 1)$

$\Rightarrow y(x) = \frac{1}{3} - \frac{\cos 3x}{3}$

QUESTION 3. (15 points) Solve for y(x), $y'(x) = e^{2x} - \int_0^x e^{2x-2r} y(r)\, dr, y(0) = 0$

$\Rightarrow y'(x) = e^{2x} - \int_0^x y(r)\, e^{2(x-r)}\, dr$ $\rightarrow y(x) * e^{2x}$

$\mathcal{L}\{y(x) * e^{2x}\} = \dfrac{Y_{(s)}}{s-2}$

Apply laplace

$\Rightarrow sY_{(s)} - y(0) = \dfrac{1}{s-2} - \mathcal{L}\{y(x) * e^{2x}\}$

$\Rightarrow sY_{(s)} = \dfrac{1}{s-2} - \dfrac{Y_{(s)}}{s-2}$

$\Rightarrow sY_{(s)} + \dfrac{Y_{(s)}}{s-2} = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)}\left(s + \dfrac{1}{s-2}\right) = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)}\left(\dfrac{s^2-2s+1}{s-2}\right) = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)}\left(\dfrac{(s-1)^2}{s-2}\right) = \dfrac{1}{s-2}$

$\Rightarrow Y_{(s)} = \dfrac{1}{(s-1)^2}$

$\Rightarrow y(x) = \mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)^2}\right\} = \underline{xe^x}$

QUESTION 4. (15 points) Solve for $x(t)$ and $y(t)$ if $x'(t) - 0.5y(t) = t$ and $x(t) + \int_0^t y(r)\, dr = 2t^2, x(0) = y(0) = 0$

① $x'(t) - 0.5\, y(t) = t$ $x(0) = y(0) = 0.$

② $x(t) + \int_0^t y(r)\, dr = 2t^2$

Apply laplace

$\mathcal{L}\{①\}$

$\Rightarrow sX_{(s)} - x(0) - 0.5(Y_{(s)}) = \dfrac{1}{s^2}$

$\Rightarrow \boxed{sX_{(s)} - 0.5\, Y_{(s)} = \dfrac{1}{s^2}}$

$\mathcal{L}\{②\}$

$\Rightarrow \boxed{X_{(s)} + \dfrac{Y_{(s)}}{s} = \dfrac{4}{s^3}}$

$X_{(s)} = \dfrac{\det\begin{bmatrix} 1/s^2 & -0.5 \\ 4/s^3 & 1/s \end{bmatrix}}{\det\begin{bmatrix} s & -0.5 \\ 1 & 1/s \end{bmatrix}}$

$= \dfrac{\dfrac{1}{s^3} + \dfrac{2}{s^3}}{1 + 0.5} = \dfrac{3}{s^3} \times \dfrac{2}{3}$

$= \dfrac{2}{s^3}$

$x(t) = \mathcal{L}^{-1}\left\{\dfrac{2}{s^3}\right\} = \boxed{t^2}$

next page

$x'(t) - 0.5\,y(t) = t$

$x'(t) = 2t$, $\Rightarrow 2t - 0.5\,y(t) = t$

$\Rightarrow -\frac{1}{2}y(t) = t - 2t$

$\Rightarrow y(t) = -2(t - 2t)$

$= -2t + 4t$

$= 2t$

QUESTION 5. (15 points) Find the general solution to $y^{(3)} + 2y^{(2)} + y' = 0$. USE THE HOMOGENEOUS METHOD.

Solution: $y = e^{mx}$

char(D.E) => $m^3 + 2m^2 + m$

set char(D.E) = 0, => $m(m^2 + 2m + 1) = 0$

=> $m(m+1)(m+1) = 0$

=> $m = 0$, $m = -1$, $m = -1$

$e^0 = 1$, e^{-x}, xe^{-x}

=> $y_g = c_1 + c_2 e^{-x} + c_3 x e^{-x}$

QUESTION 6. (20 points) a) Find the general solution to $y^{(2)} + 16y = 0$

Solution: $y = e^{mx}$

char(D.E): $m^2 + 16$

set char D.E = 0: $m^2 + 16 = 0$

=> $m^2 = -16$

=> $m = \pm\sqrt{-16}$

=> imaginary => use one solution

$+\sqrt{-16} = a + bi$

=> $\sqrt{-16} = a + bi$

=> $a = 0$, $b = \sqrt{16} = 4$

=> $y_1 = e^{ax} \cos bx = \cos 4x$

$y_2 = e^{ax} \sin bx = \sin 4x$

=> $y_G = c_1 \cos 4x + c_2 \sin 4x$

b) If $y(0) = 0$ and $y'(\pi/8) = 0$, what will be the solution of the D.E in part (a)? Does that contradict one of the Theorem in the book? EXPLAIN (Note $sin(\pi/2) = 1, cos(\pi/2) = 0, sin(0) = 0, cos(0) = 1$)

$y(x) = c_1 \cos 4x + c_2 \sin 4x$

=> $y(0) = 0 = c_1$

=> $c_1 = 0$

$y'(\frac{\pi}{8}) = -4c_1 \sin 4x + 4c_2 \cos 4x$

$0 = -4c_1$

=> $c_1 = 0$

$y_a = c_2 \sin 4x$

next page

c) If $y(0) = 0$ and $y'(\pi/8) = 1$, what will be the solution of the D.E. in part (a)? Does that contradict one of the Theorem in the book? EXPLAIN

* has to be same x value.

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a)

$$y'' + 16y = 0$$

Apply laplace $\Rightarrow s^2 Y(s) - s y(0) - y'(0) + 16 Y(s) = 0$

$$\Rightarrow Y(s)\left[s^2 + 16\right] = s y(0) + y'(0)$$

$$\Rightarrow Y(s) = \frac{s y(0) + y'(0)}{s^2 + 16}$$

$$\Rightarrow Y(s) = \frac{s c_1}{s^2+16} + \frac{c_2}{s^2+16}$$

$$= c_1 \cos 4x + c_2 \sin 4x$$

b) $y(0) = 0$

$y'(\pi/8) = 0$.

$$\Rightarrow Y(s) = \frac{y'(0)}{s^2 + 16}$$

$$\Rightarrow y(x) = \frac{c_1}{4} \sin 4x$$

$$y'(x) = c_1 \cos 4x$$

$$y'\left(\tfrac{\pi}{8}\right) = c_1 (0)$$

$$= 0$$

$\Rightarrow$ no contradiction

c) $y'\left(\frac{\pi}{8}\right) = 0$, can't be $= 1$

there is a contradiction since $1 = c(0)$

$\Rightarrow c = \frac{1}{0}$ $\Rightarrow$ no constant can be multiplied to satisfy the theorm

$\Rightarrow$ contradiction.

ESE/G

OK

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