TEST NUMBER TWO FOR MATH 221, FALL 2004

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Name——————————, Id. Num.————————————, Score $\frac{}{100}$

QUESTION 1. *(***20 POINTS***) (***True or False***)*

(1) *Let A be a 4×5 such that $Rank(A) = 3$. Then any three columns of A are independent.*
(2) *Let A be a 3×6 such that $Rank(A) = 3$. Then $AX = b$ has a solution for every b , 3×1.*
(3) $Span\{1 + x, 2x + x^2, -3x^2\} = P_3$.
(4) $S = \{(x, y) \in R^2 \mid y = 3x + 1\}$ *is a subspace of* R^2.
(5) *The span of any 5 elements in R^5 is equal to R^5.*
(6) *It is possible that the span of 6 elements in $R^{2\times2}$ is equal to $R^{2\times2}$.*
(7) *If A is 6×8 and $AX = b$ has no solution for some b, 6×1, then the column space of A is NOT equal to R^6.*
(8) *The interval $(-\infty, 300)$ is a subspace of R.*
(9) *It is possible to construct a 6×5 matrix with rank equals to 6.*
(10) $span\{(1, 0, 2), (0, 4, 10)\} = R^3$.

QUESTION 2. *(***9 POINTS***) Let $S\{f(x) \in P_4 \mid f(x) = a + (a + b)x + bx^2 + (2a - 3b)x^3\}$ be a subspace of P_4. What is the dimension of S? Find a basis for S.*

QUESTION 3. *(**8 POINTS**) Let* $S = \{f(x) \in C[-2,2] \mid f(1) = 0$ *OR* $f(-1) = 0\}$*. Is* S *a subspace of C[-2, 2]? EXPLAIN*

QUESTION 4. *(**13 POINTS**) Let* $S = \{A \in R^{2\times} \mid a_{11} + a_{22} = 0$ *and* $a_{12} + a_{21} = 0\}$*. Show that* S *is a subspace of* $R^{2\times 2}$*, and then find a basis for* S*.*

QUESTION 5. *(***8 POINTS***) Find a basis for* P_4 *that contains the two independent elements:* $1 + x + x^2$ *and* $-1 - x + x^3$*. Show the steps.*

QUESTION 6. *(***9 POINTS***) Given that* $(-2, 0, 2) \in Span\{(-1, 1, 1), (3, 1, -3)\}$*. Find* α_1 *and* α_2 *such that* $(-2, 0, 2) = \alpha_1(-1, 1, 1) + \alpha_2(3, 1, -1)$*.*

QUESTION 7. *(***8 POINTS***) Is* $Span\{(1, -1, 2), (-1, 1, 0), (-1, -1, -2), (-1, 1, 2)\} = R^3$*? EXPLAIN*

QUESTION 8. *Let* $A = \begin{bmatrix} -1 & 0 & 1 & -1 & -1 \\ 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

(1) *(**15 POINTS**)Find the* $N(A)$*, Nullity of (A), and a basis for* $N(A)$*.*

(2) *(**5 POINTS**)Find a basis for the column space of (A)*

(3) *(**5 POINTS**)Find a basis for the row space of* A *.*

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