MATH 221, FIRST EXAM, SPRING 2004

QUESTION 1. *Write down true or false. If false, then give a counter example: (10 points)*

(1) *If* A *is an* $n \times n$ *matrix and not invertible, then the reduced echelon form of* A *has at least one row consists of zeros (* *)*

(2) *If* A *is a* 3×3 *matrix and* $det(A^{-1}) = 2$*, then* $det(A^T) = 1/2$ *(* *)*

(3) *If* A, B *are a* 4×4 *matrices and* A *is row-equivalent to* B*, then* $det(A) = det(B)$*. (* *)*

(4) *If a homogeneous system has infinitely many solutions, then the system has more variables than equations.*

(5) *If* A *is* 3×3 *and* $AX = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$ *has no solution, then* $det(A) = 0$ *(* *)*

QUESTION 2. *(15 points)*
Given A *is a* 5×5 *matrix and* $det(adj(A)) = 16$

a) Find $det(3A^{-1})$

b) Find $det(2A^T)$

c) Find $det(I_3 + Aadj(A))$.

QUESTION 3. *Consider the following system*
$2x_1 - 2x_2 + 4x_3 - 2x_4 = -2$

$-x_1 + 2x_2 + x_3 + 2x_4 = 2$
$x_1 + x_2 + 4x_3 + 3x_4 = 3$

a) Write the above system in the form $AX = B$.*(5 points)*

b) Find the general solution for $AX = B$. *(10 points)*

C) Find the general solution for $AX = 0$*(5 points)*

QUESTION 4. *(24 points)*

a) Given A A_1 A_2 $\begin{bmatrix} 2 & 1 & 1 \\ -2 & -2 & 0 \\ -3 & 5 & 6 \end{bmatrix}$. *Find* $det(A)$.

b)Let $A = \begin{bmatrix} 2 & 3 & -1 & 0 \\ 1 & -3 & -2 & 3 \\ -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & 4 \end{bmatrix}$ *Find the (3,2)-entry of* A^{-1}.

c)Let A, B *be* 3×3 *matrices such that*

A A_1 B. *Find two elementary matrices* E_1, E_2 *such that* $A = E_1 E_2 B$.

d) Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 0 & 3 \\ 0 & x & -2 \end{bmatrix}$. *Find the value of* x *that will make* A *invertible.*

QUESTION 5. *Let* A, B *be nonzero* $n \times n$ *matrices such* $AB = 0$. *Prove that neither* A *nor* B *is invertible.*