MATH 221, THIRD EXAM, SUMMER 008

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QUESTION 1. *(20 points)* **Let A be a 3×3 matrix such that**

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$A \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = 2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$A \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = -2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

a) Explain why A is nonsingular.

b) Find A^{-1}.

QUESTION 2. 20 points *Let* $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

a) Find all eigenvalues of A.

b) If A is diagnolizable, then find a diagonal matrix D and a nonsingular matrix Q such that $Q^{-1}AQ = D$.

QUESTION 3. 20 points *a) Show that* $D = \{f(x) \in P_3 \mid \int_0^1 f(x)\, dx = 0\}$ *is a subspace of* P_3.

b) Find a basis for D then Rewrite D as span.

c) Is $L = \{f(x) \in P_2 \mid f(1) = 0 \ OR \ f(-1) = 0\}$ *a subspace of* P_2? *Explain. If Yes, then find a basis.*

QUESTION 4. 20 points *1)Find the LU-factorization of* $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 2 & 1 \\ -2 & -2 & 3 & 4 \\ -1 & -1 & -1 & 6 \end{bmatrix}$

2) Given A is 3×3 *matrix and* $A \xrightarrow{2R_1} A_1 \xrightarrow{-3R_1 + R_2 \to R_2} A_2 \xrightarrow{R_2 \leftrightarrow R_3} I_3$.

a) Find A.

b) Explain WHY A is nonsingular.

c) Write A^{-1} *as product of elementary matrices.*

QUESTION 5. 20 points *Let A be a 3×3 a nonsingular matrix with -2, -1, 2 as eigenvalues.*

a) Is $A + 4I_3$ nonsingular? EXPLAIN

b) Find the eigenvalues of $adj(A)$

(c) Find $det(A^{-1} + 2I_3)$

QUESTION 6. *a) If $2, 9, 28$ are the eigenvalues of the matrix $(A^3 + I_3)$. Find $det(A)$ (observe that A is 3×3)*

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