MATH 221, SECOND EXAM, SUMMER 008

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QUESTION 1. *(***20 points) Write DOWN T OR F**

(1) **If** A **is a** 2×2 **matrix and** $Char(A) = x^2 + 8x + 6$**, then** $V = 0$ **(the zero-vector in** R^2**) is the only solution for** $AV = 4V$**.**

(2) **If** A **is** 5×5 **and singular, then the system** $AX = 0$ **has infinitely many solution.**

(3) **If** A $(n \times n)$ **is singular, then it is possible that** A^T **is nonsingular.**

(4) **The span of every 3 vectors in** R^3 **is equal to** R^3**.**

(5) **dim(**$Span\{(2,2,5), (-2,-2,6), (2,2,16)\}) = 2$

(6) **If** A **is** 23×23 **matrix and** $A \times adj(A)$ **is the zero-matrix, then**$det(A) = 0 = det(adj(A)) = 0$**.**

(7) **Suppose** A **is** 5×3 **and the system** $AX = 0$ **has a unique solution. Then if** b **in** R^5 **(b written as a column) and AX = b is consistent, then** $AX = b$ **has a unique solution.**

(8) **If** A **is singular and** B **is nonsingular, then** AB **is singular.**

(9) **If** A, B **are nonsingular** $n \times n$ **matrices, then AB is nonsingular and** $(AB)^{-1} = A^{-1}B^{-1}$**.**

(10) **If** A **is a** 3×6 **matrix and** $A \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 7 \end{bmatrix}$**, then the system** $AX = \begin{bmatrix} 0 \\ 6 \\ 7 \end{bmatrix}$ **has infinitely many solutions.**

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QUESTION 2. (**6points) Let** A **be a** 2×2 **matrix such** $adj(A) = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$**. Find** A**, then find** $det(A)$**.**

QUESTION 3. (**8 points) Let** A **be a** 6×8 **matrix and** b **be the fourth column of** A**. Show that the system** $AX = b$ **has infinitely many solutions. Give me one particular solutions.**

QUESTION 4. (**6 points) Let** $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$**. Assume** A **is nonsingular. Show that the (2, 3)-entry of** A^{-1} **is equal to** $det(B)/det(A)$ **where** $B = \begin{bmatrix} a_{11} & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & 1 & a_{33} \end{bmatrix}$**.**

QUESTION 5. *(***20 points) Let** $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ -1 & -1 & 2 & -1 & 4 \\ 2 & 2 & 2 & 2 & 5 \\ 1 & 1 & 1 & 1 & 2 \\ -1 & -1 & -1 & -1 & -2 \end{bmatrix}$.

a) Find Rank (A), find basis for Row(A), then write ROW(A) as a span.

b)Find a basis for Col(A).

c) Find $N(A)$**, basis for N(A), then write N(A) as a span.**

QUESTION 6. *(***6 points) Find a basis for** R^4 **containing the two independent vectors,** $v_1 = (6, -4, 6, 6)$**, and** $v_2 = (-12, 8, -12, -11)$

QUESTION 7. *(***6 points) For what values of** x **will the following matrix be nonsingular:**

$$A = \begin{bmatrix} x+5 & 0 & 3 \\ 2 & 0 & x \\ 12 & x-2 & 300 \end{bmatrix}$$

QUESTION 8. *(***8 points) Find** A^{-1} **if** $A = \begin{bmatrix} 1 & -2 & 4 \\ -1 & 2 & -3 \\ -2 & 5 & -8 \end{bmatrix}$

QUESTION 9. *(***20 points) Let** $A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ -2 & -2 & -2 & 6 \\ -1 & 2 & 7 & 9 \\ -4 & -4 & 4 & 4 \end{bmatrix}$

a)Use row operations to find $det(A)$**.**

b) Find the (2,3)-entry of A^{-1}**.**

c) Let V **be the third column of** A^{-1}**. Calculate** AV**.**

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