FINAL EXAM FOR MATH 221, FALL 2004

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Name——————————, Id. Num.————————————, Score $\frac{}{100}$

QUESTION 1. *(***10 POINTS***) (***True or False***)*

(1) *If* A *is a* 4×4 *nonsingular matrix, then* $Rank(A) = 4$.
(2) *If* A *is a* 4×4 *matrix, then* $det(A) = det(-A)$.
(3) $\{(x, y) \in R^2 \mid x \geq 0\}$ *is a subspace of* R^2.
(4) *If* A *is a* 3×4 *matrix, then there is a* b, 3×1, *such that* $AX = b$ *has a unique solution.*
(5) $Span\{(1 + x^2, x^2\} = P_3$

QUESTION 2. (1) *(***10 points***) Let* $T : R^4 \longrightarrow R^3$ *be a linear transformation such that* $T(x_1, x_2, x_3, x_4) = (x_1 + x_2, -x_1 - x_2 + x_3, -x_1 + 2x_3)$. *Find a basis for the Kernel of* T *and a basis for the Range (Image) of* T.

(2) (**4 points**) *Let* $T : R^2 \longrightarrow R$ *such that* $T(2,-4) = -1$ *and* $T(-2,1) = 2$. *Find* $T(6,-9)$.

(3) (**6 points**) *If dim(* $S = span\{(-1,-1,0,-1),(-1,-1,1,-1),(0,-1,0,1)\}) = 3$. *Find an orthonormal basis for* S *using Gram- Schmidt-Process.*

QUESTION 3. *(**15 points**) Find the eigenvalues and the corresponding eigenspaces of the matrix* $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. *Given that* A *is similar to a diagonal matrix* D. *Find* D *and find a matrix* Q *such that* $Q^{-1}AQ = D$.

QUESTION 4. *(**6 points**) Given that* A, B *are* 4×4 *matrix such that* $det(A) = -2$ *and* $det(B) = 1/2$. *Find*

(1) $det(2AB^T)$

(2) $det(A^{-1}B)$

(3) $det(B^{-1} + adj(B))$

QUESTION 5. *(**6 points**) Let* $A = \begin{bmatrix} a & -2 & -2 \\ b & 4 & -1 \\ 5 & 0 & 0 \end{bmatrix}$. *Show that* $det(A) \neq 0$. *Then find the* $(1,3)$*-entry of* A^{-1}.

QUESTION 6. (1) *(**6 points**) Given that* $S = \{(x_1, x_2, x_3, x_4) \in R^4 | x_1 = x_2 - 3x_3$ *and* $x_4 = 2x_2 - x_3\}$ *is a subspace of* R^4. *Find the dimension of* S, *then find a basis for* S.

(2) (**8 points**) *Let* $S = \{A \in R^{2\times 2} \mid A \text{ is an upper triangular matrix}\}$. *Show that* A *is a subspace of* A. *Then find a basis for* S.

(3) (**4 points**) *Let* $S = span\{1+2x+x^2, -1+x+x^2, 3x+2x^2\}$. *Find a basis for* S.

(4) (**4 points**) *Is* $S = \{A \in R^{2\times 2} \mid det(A) = 0\}$ *a subspace of* $R^{2\times 2}$? *Explain*

QUESTION 7. (**6 points**) *Given* A, B *are* 3×3 *matrices such that* B *is obtained from* A *by performing the following two row-operations on* A:
$A \xrightarrow{2R_1 + R_3 \to R_3} A_1 \xrightarrow{3R_3} B$. *Find two elementary matrices* E_1, E_2 *such that* $A = E_1 E_2 B$.

QUESTION 8. (**6 points**) *Let* $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ -2 & 2 & 1 \end{bmatrix}$ *Find* A^{-1}.

QUESTION 9. *(***9 points***) Given the augmented matrix of a system of linear equations:*

$$\begin{bmatrix} 1 & 2 & a & b \\ -1 & -1 & 2 & -2 \\ 1 & 2 & c & 6 \end{bmatrix}$$

(1) *Find the values of* a, b, c *so that the system will have a unique solution.*
(2) *Find the values of* a, b, c *so that the system will have infinitely many solutions.*
(3) *Find the values of* a, b, c *so that the system will have NO SOLUTION*

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