MTH221, LINEAR ALGEBRA, FINAL EXAM FALL 2006

Question 1. (**6 points**) *Let* $A = \begin{bmatrix} 2 & 123 & -1 \\ 1 & 456 & 1 \\ 2 & 789 & 1 \end{bmatrix}$ *If you know that* $det(A) = -420$, *then find the value of* x_2 *in the solution of the linear system* $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Question 2. *Determine whether the following vectors are linearly independent in the vector space V* (**SHOW YOUR WORK).**

a)(4 points) $V = R^3$, $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

b) (4 points) $V = P_3, \quad 3x^2, \quad x^2 - 10x + 15, \quad 10x - 15$

Question 3. *For each of the following sets of vectors, determine if it is a subspace. If YES explain why and, if your answer is a NO then give an example to show why it is not subspace.*

*(a) (***5 points***)* $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_1 - 5x_3 = 0 \right\}$

*b)(***5 points***)* $S = \{B \in R^{2\times 2} \mid A \ \ \textit{is singular (non-invertible)} \ \}$

Question 4. *Find a Basis for each of the following subspace S of the given vector space V. (***DO NOT SHOW IT IS A SUBSPACE***)*

*a) (***5 points***)* $V = R^5$, $S = \left\{ \begin{bmatrix} a+b \\ b \\ c \\ 0 \\ c+b \end{bmatrix} \mid a, b, c \in R \right\}$

*b) (***5 points***)* $V = R^{2x2}$, $S = \{A \in R^{2\times 2} \mid -A = A^T\}$

*c)(***5 points***)* $V = P_4$, $S = \{p \in P_4 \mid p(1) = p(0) = 0\}$

Question 5. *Let* $S = Span\left\{ \begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\2\\2\end{bmatrix} \right\}.$

*a)(***4 points***) Find a basis for S.*

*b)(***4 points***) Find an ORTHONORMAL BASIS for S.*

Question 6. *Consider the following set of polynomials in* P_3.

$$S = \{t^2 + 1,\ t^2 + 2t,\ 3t^2 + t - 1\}$$

c. *(***4 points***) Find a basis for Span*$\{S\}$.

b. *(***4 points***) Does* $6t^2 - 1$ *belong to Span*$\{S\}$*?*

Question 7. *We have a* 3×3 *matrix* $A = \begin{bmatrix} a & 1 & 2 \\ b & 3 & 4 \\ c & 5 & 6 \end{bmatrix}$ *with* $det(A) = 3$. *Compute the determinant of the following matrices:*

*(a) (***2 points***)* $\begin{bmatrix} a-2 & 1 & 2 \\ b-4 & 3 & 4 \\ c-6 & 5 & 6 \end{bmatrix}$

*(b) (***2 points***)* $\begin{bmatrix} 7a & 7 & 14 \\ b & 3 & 4 \\ c & 5 & 6 \end{bmatrix}$

*(c) (***3 points***)* $2A^{-1}A^T$

*d) (***4 points***)* $\begin{bmatrix} a-2 & 1 & 2 \\ b & 3 & 4 \\ c & 5 & 6 \end{bmatrix}$

Question 8.

$$If\ A = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$$

*a) (***4 points***) Find all eigenvalues A*

*b) (***6 points***)Find a nonsingular matrix* Q *and a diagonal matrix* D *such that* $Q^{-1}AQ = D$ *(that it is* $A = QDQ^{-1}$*)*

*c) (**5 points**) For the matrix A in Question number 8, find A^5*

Question 9. (**6 points**) *Let* $L : R^3 \longrightarrow R^3$ *be a linear transformation such that*

$$L\left(\begin{bmatrix}-2\\1\\-2\end{bmatrix}\right) = \begin{bmatrix}-3\\1\\2\end{bmatrix}, \quad L\left(\begin{bmatrix}3\\2\\-1\end{bmatrix}\right) = \begin{bmatrix}2\\-2\\1\end{bmatrix}, \quad L\left(\begin{bmatrix}-1\\-1\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\2\\4\end{bmatrix}.$$

Find $L\left(\begin{bmatrix}1\\1\\-1\end{bmatrix}\right)$

Question 10. *It is given that* $A = \begin{bmatrix} 1 & 0 & -2 & 1 & 3 \\ -1 & 1 & 5 & -1 & -3 \\ 0 & 2 & 6 & 0 & 1 \\ 1 & 1 & 1 & 1 & 4 \end{bmatrix}$ *and its reduced row echelon form is given by* $B = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) *(***2 points***) Find the rank of A.*

(b) *(***2 points***)Find the nullity of A.*

(c) *(***3 points***) Find a basis for the column space of A.*

(d) *(***3 points***) Find a basis for the row space of A.*

(e) *(***3 points***) Find a basis for the null space of A.*