Question 1. *Write down true or false (***12 points***)*

1. (1) *If A is an $n\times n$ matrix and b is a column matrix, $n\times 1$, such that $AX = b$ has infinitely many solutions, then 0 is an eigenvalue of A.*
2. (2) *If A is an $n \times n$ matrix, then A and A^T have the same eigenvalues.*
3. (3) *If* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *, then the adjoint of A is* $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
4. (4) *If A, B are row equivalent $n \times n$ matrices, then both have the same eigenvalues.*
5. (5) *If a homogeneous system has infinitely many solutions, then the system has more variables than equations.*
6. (6) *If A is a 3×4 matrix with a rank equals to 2, then there must be a vector b, 3×1, such that $AX = b$ has no solution.*

Question 2. (1) *(***6 points***) Use Cramer's method to find the solution for x_2*

$$2x_1 - 2x_2 + 4x_3 = 2$$
$$-2x_1 + x_2 + x_3 = 1$$
$$x_1 - x_2 + 3x_3 = 2$$

(2) *(***8 points***)Given $dim(span\{(-1, 1, 0, 1), (1, 1, 1, 1), (1, -1, 0, 1)\}) = 3$. Find an orthonormal basis for S.*

Question 3. (1) *(***4 points***)Let A be a 6×4 matrix such that the second column of A and the fourth column of A are identical, and let b be a 6×1 column matrix. Given that* $X = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 3 \end{bmatrix}$ *is a solution to the system $AX = b$. Find another two different solutions to the system $AX = b$.*

(2) *(***6 points***)Given A is a 3×3 matrix such that*

$$A \underbrace{3R_2} A_1 \underbrace{-2R_2 + R_3 \to R_3} B = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$$

a) Find $det(A)$

b) Find two elementary matrices E_1 and E_2 such that $A = E_1 E_2 B$

(3) *(***6 points***)Given the augmented matrix of a system of linear equations*

$$A = \begin{bmatrix} -2 & 3 & a & 3 \\ 2 & -2 & b & -2 \\ 2 & -3 & 4 & c \end{bmatrix}$$

1) For what values of a, b, c does the system have unique solution?

2) For what values of a, b, c does the system have no solution.

3) For what values of a, b, c does the system have infinitely many solutions?

Question 4. (1) *Let* $A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

*(***14 points***) Find a nonsingular matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.*

(2) *(***4 points***) Let* $A = \begin{bmatrix} 3 & a \\ 0 & 3 \end{bmatrix}$*. For what values of a is the matrix A diagonalizable? EXPLAIN.*

Question 5. (1) (**8 points**) *Let* $A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ *and let* $S = \{B \in R^{2\times 2} \mid AB = BA\}$. *Show that* S *is a subspace of* A. *Find a basis for* S.

(2) (**4 points**) *Given* $S = \{f(x) \in P_5 \mid f(x) = (b+c) + bx + (2b-d)x^2 + cx^3 + dx^4\}$ *is a subspace of* P_5. *Find a basis for* S.

(3) (**6 points**) *Does* $(0, 1, 2, 3)$ *belong to* $S = span\{(1, -1, 0, 1), (-1, 2, 1, 1), (-1, 1, 1, 0)\}$? *EXPLAIN*

(4) (**4 points**) *Let* $S = \{(a, b, c) \in R^3 \mid a + b + c + 1 = 0\}$. *Is* S *a subspace of* R^3? *if No, then explain. If yes, then find basis for* S.

Question 6. *Let* $A = \begin{bmatrix} 1 & -1 & 2 & -2 \\ -2 & 2 & -4 & 5 \\ -3 & 3 & -6 & 6 \end{bmatrix}$ *Given* $T : R^4 \longrightarrow R^3$ *such that* $T(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}) = A \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$ *is a linear transformation*

a) (**6 points**) *Find a basis for* $Ker(T)$

b)(**6 points**) *Find a basis for Range(T).*

(**6 points**) *Let* $T : R^2 \longrightarrow R^3$ *be a linear transformation such that* $T(2, -4) = (1, 1, 1)$ *and* $T(-3, 8) = (-2, -2, 3)$. *Find the standard matrix representation of* T.

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