FINAL EXAM MTH 221, SPRING006

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QUESTION 1. *Let* $T : P_3 \longrightarrow R^3$ *be a linear transformation such that* $T(f(x)) = (f'(0), f(1), 0)$.
a) Show that T *is a linear transformation. (***4 points***)*

*b) Find basis for Ker(T) (***6 points***).*

*c)Find basis for Range(T) (***4 points***).*

QUESTION 2. *Let* $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$

*1)Find the eigenvalues of A. (***4 points***)*

*2)For each eigenvalue of A find the corresponding eigenspace.(***10 points***)*

Question TWO continues: 3) Find nonsingular matrices Q_1, Q_2 *AND diagonal matrices* D_1, D_2 *such that* $Q_1^{-1}AQ_1 = D_1$ *and* $Q_2^{-1}AQ_2 = D_2$.*(***5 points***)*

QUESTION 3. *Let* $S = Span\{(1,0,1,-2),(0,1,0,2),(-2,1,-2,6)\}$.
*1) Find a basis for S. (***4 points***)*

*2)Find an orthogonal basis for S.(***4 points***)*

QUESTION 4. *The following are subspaces (DO NOT NEED TO SHOW THAT). FIND A BASIS FOR EACH ONE OF THEM:*

1) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$, *and let* $S = \{B \in R^{2\times 2} \mid AB = BA\}$ (**6 points**)

2) Let $S = \{f(x) \in P_4 \mid f(1) = f(-1)\}$. (**6 points**)

QUESTION 5. *Does* $span\{1 + x^2, x^2, 2 + 5x^2\} = P_3$ *(explain)* (**4 points**)

QUESTION 6. *Given* A *is a* 3×3 *matrix such that* $A \underbrace{2R_2} B \underbrace{-R_1 + R_2 \to R_2} C = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 2 & -2 \\ 0 & 1 & 1 \end{bmatrix}$. *a) Find an elementary matrix* E *such that* $A = EB$. *(***3 points***)*

b) Find the matrix A. *(***5 points***)*

QUESTION 7. *Let* $A = \begin{bmatrix} 2 & -2 & -3 & 1 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. *Find the (2, 3)-entry of* A^{-1}. *(***5 points***)*

QUESTION 8. *1)Find a basis for R^4 that contains the following two independent elements :* $(-4, 0, 1, 6), (4, 0, -1, -4)$. *(***4 points***)*

QUESTION 9. *Let* $L : R^2 \longrightarrow R$ *be a linear transformation such that* $L(2, 2) = -4$ *and* $L(2, 1) = -6$. *Find* $L(10, 8)$ *(***5 points***)*

QUESTION 10. *Let* $A = \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & 0 \\ 2 & -2 & -2 & 2 \end{bmatrix}$

1) Solve $AX = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$ *(***5 points***)*

2) Find a basis for the column space of A. (4 points)

QUESTION 11. *1) Given A is a 3×3 matrix with eigenvalues 4, -2, -3.*
a) Find the eigenvalues of $A^2 + 5I_3$. (**4 points)**

b)What is the $det(A^{-1})$ (2 points).

QUESTION 12. *Let A be a 3×3 matrix such that* $(A^{-1} + 2I_3)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 0 & 4 \end{bmatrix}$.
Find the matrix A. (**6 points**)

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