MATH 221, FINAL EXAM (9:30), SUMMER 008

AYMAN BADAWI

QUESTION 1. **(15 points) (Write DOWN T OR F)**

(1) **If V is an eigenvector of a matrix $n \times n$ A, then V is also an eigenvector of A^3.**

(2) **If A is 4×3, then the homogeneous system $AX = 0$ has infinitely many solutions.**

(3) **If A is nonsingular, 3×3 , then the reduced echelon form of A is I_3.**

(4) **If A is a 3×5 matrix such that $Nullity(A) = 2$, then for every column matrix 3×1 b the system $AX = b$ has infinitely many solutions.**

(5) **If 2 is an eginvalue of a 4×4 matrix A, then the matrix $A - 2I_4$ is singular.**

(6) **If A is a 4×4 matrix, then $det(3A) = 3det(A)$.**

(7) **If 7 is an eigenvalue of an $n \times n$ matrix A, then $\frac{1}{7}$ is an eigenvalue of A^T.**

(8) **If A is an 3×3 matrix and $A = A^{-1}$, then $A^3 = A$**

(9) **If A is nonsingular, then A^T is nonsingular.**

(10) **At this minute, you are in Nab 10.**

QUESTION 2. **(5 points) Find the matrix 2×2 A such that**

$$\left(A \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\right)^{-1} = A^{-1} + 2I_2$$

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QUESTION 3. (**12 points) Let** $A \begin{bmatrix} 3 & 0 & -2 \\ -2 & 2 & 4 \\ 1 & 0 & 0 \end{bmatrix}$ **For each eigenvalue of** A **find the corresponding eigenspace. If** A **is diagnolizable, then find a diagonal matrix** D**, and a nonsingular matrix** Q **such that** $Q^{-1}AQ = D$**.**

QUESTION 4. *Given* $T: R^3 \longrightarrow R^4$ *is a linear transformation such that*

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\-1\\1\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\4\\-2\\2\end{bmatrix}, \textit{ and } T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}4\\8\\3\\4\end{bmatrix}$$

*a) (***5 points) Find** $T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)$

b) (8 points) Find the standard matrix representation of T. Then find a basis for Ker(T), and a basis for Range(T).

QUESTION 5. *(***6 points) Let** $A = \begin{bmatrix} 2 & -2 & 3 \\ -1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$**. Find** A^{-1}**, and find** $(A^T)^{-1}$**.**

QUESTION 6. *Given* $A \quad \overrightarrow{R_1 \leftrightarrow R_2} \quad A_1 \quad \overrightarrow{16R_2} \quad A_2 \quad \overrightarrow{-R_1 + R_3 \to R_3} \quad A_3 = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & -3 \\ 0 & 0 & 6 \end{bmatrix}$.

*a)(***4 points) Find** $det(2A)$**.**

b)(5 points) Write A **as product of elementary matrices with** A_3**.**

QUESTION 7. *a)* (**5 points**) $D = \{f(x) \in P_4 \mid f'(0) = 0, f(1) = 0\}$ **is subspace of** P_4 **(DO NOT SHOW THAT). Find a basis for** D**.**

b)(5 points) $L = \{A \in M_{3\times 2}(R) \mid a_{21} + a_{22} + a_{31} = 0\}$ **is a subspace of** $M_{3\times 2}(R)$ **(DO NOT SHOW THAT). Find a basis for** L**.**

c) (5 points) Let $M = span\{1 + x, -1 + 2x, x^2, 3x + x^2\}$**. Find a basis for M.**

QUESTION 8. *a) (***6 points) Use CRAMER RULE to solve for** x_3

$x_1 + x_2 + x_3 = 12$

$-x_1 + x_2 - x_3 = -10$

$-5x_1 + -5x_2 + 2x_3 = 10$

b)(6 points) Let $A = \begin{bmatrix} a & 3 & 1 \\ b & 5 & 2 \\ c & 1 & 3 \end{bmatrix}$ **and the REDUCED ECHELON FORM OF** A **IS the MATRIX** $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$**. Find** a, b, c **(SHOW THE WORK....GUESSING IS NOT ACCEPTABLE)**

QUESTION 9. *a)(***7 points) Solve the following system**

$x_1 + 4x_2 + 3x_3 - x_4 + x_5 = 2$

$-x_1 - 3x_2 - 3x_3 + x_4 + x_5 = -4$

$2x_1 + 8x_2 + 6x_3 - x_4 + 3x_5 = -2$

b) (6 points) Let $D = Span\{1, 0, 1, 2), (1, 1, 0, 0), (1, 0, -1, -1)\}$**. Find an ORTHOGONAL BASIS for** D**.**

Department of Mathematics & Statistics, American University Of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates, www.ayman-badawi.com

E-mail address: abadawi@aus.edu