MTH 213, FINAL EXAM , SPRING 006

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Name————————-, Id. Num.————————————-, Score $\frac{}{100}$

QUESTION 1. *(**6 points**) Prove that* $((\neg p) \to q) \leftrightarrow p$ *is a tautology statement.*

QUESTION 2. *(**4 points**) Assume that* p *represents the statement " Ayman is happy," q represents the statement "Nada is in pain," and r represents the statements "Ali is angry." Write the following statements using LOGICAL CONNECTIVES:*

(1) *If Ali is angry, then either Ayman is happy or Nada is in pain.*

(2) *Nada is in pain if and only if Ayman is happy and Ali is not angry.*

QUESTION 3. *(**28 points**) T OR F*

(1) $\{3\} \in \{3, \{3\}, \{3, \{3\}\}\}$.

(2) *if* $S = \{3, \{3\}, \{3, \{3\}\}\}$, *the* $P(S)$ *(Power set of S) has exactly 16 elements.*

(3) *If* $S = \{3, \{4\}, 4, \{6\}\}$, *then* $\{3, \{4\}\} \in P(S)$.

(4) C_{13} *is a bipartite graph.*

(5) $C_{2,37}$ *has an Euler path but not an Euler circuit.*

(6) *Let* $x \in Z^+, y \in Z$, *and* $Q(x,y)$ *be the statement :* $x^2 + y = 28$ *:*
   (a) $\forall x\ \exists y\ Q(x,y)$.

   (b) $\exists y\ \forall x\ Q(x,y)$.

   (c) $\exists!\ x\ Q(x,24)$

(7) *It is possible to construct a complete graph with 11 vertices and 45 edges.*

(8) $a_n = 7a_{n-3}$ *has* $C(x) = x - 7$ *as the characteristic polynomial.*

(9) *It is possible to construct a graph with 7 vertices such that each vertex has degree 3.*

(10) *-54 mod (7) = 6*

(11) *When dividing -96 by 22, the quotient (q) is 4*

(12) *Let* $n \in Z^+$. *If* 15 *divides* n *and* 14 *divides n, then (15)(14) divides n.*

QUESTION 4. *(***6 points***) Let* P *be the set of all positive prime integers,* $x \in P$, $y \in Z^+$, *and* $Q(x,y)$ *be the statement: x divides y. Rewrite the following statement using QUANTIFICATION:*

(1) *3 divides y and 5 divides y.*

(2) *there are many prime numbers that do not divide 24.*

(3) *none of the prime numbers divides 1.*

QUESTION 5. *(***5 points***) Let* $a_n = da_{n-1} + fa_{n-2}$. *If* $a_n = (3+7n)5^n$, *find d and f.*

QUESTION 6. *(***10 points***) Given* $a_n = 4a_{n-2} + 42^n$ *such that* $a_0 = -6$, $a_1 = 10$. *Find a mathematical equation for* a_n.

QUESTION 7. *(***10 points***) Describe all NEGATIVE INTEGERS that have the following properties: if each is divided by 7, then the remainder is 4. If each is divided by 4, then the remainder is 3. If each is divided by 9, then the remainder is 7.*

QUESTION 8. *(***10 points***) Let* $a = 315$, $b = 231$. *Find the* $gcd(a, b)$. *Write the* $gcd(a, b)$ *as a linear combination of* a *and* b.

QUESTION 9. *(**8 points**) Let* $n = 21.6^3.33.55,\ \ m = 3^5.5^{10}.44.$

1. *Find the* $LCM[n, m]$.

2. *Find* $\phi(n)$.

QUESTION 10. *(**5 points**) Let* G *be a simple graph with 100 vertices such that each vertex has degree 6.*
1) What is the degree of each vertex of the graph $\overline{G}$*?*

2) How many edges does $\overline{G}$ *have?*

QUESTION 11. *(***8 points***) Find the shortest path from* V_1 *to* V_{10}

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