

FINAL EXAM FOR MTH213 SPRING007

AYMAN BADAWI

Name _____, Id. Num. _____, Score $\overline{100}$

QUESTION 1. (12 points) Let $a = 7^2 \cdot 12 \cdot 3 \cdot 5^4$ $b = 2^4 \cdot 15^2 \cdot 21^3$

a) Find the $LCM[a, b]$

b) Find the $gcd(a, b)$

c) Find $\phi(b)$.

d) Let $F = \{k \mid 1 < k < b \text{ and } gcd(k, b) = 3\}$ Find $\sum_{k \in F} k$.

QUESTION 2. (10 points) Find the *LARGEST THREE NEGATIVE INTEGERS* with the following properties: If each is divided by 7, the remainder is 3. If each is divided by 9, then the remainder is 6.

QUESTION 3. (5 points) Find the least positive integer a such that $5^{243} \equiv a \pmod{99}$.

QUESTION 4. (12 points) a) Given $b \in \mathbb{Z}^+$ such that $b \equiv 15 \pmod{22}$. Find the $\gcd(22, b)$ Explain.

b) If $b \in \mathbb{Z}^+$ and $b \equiv -7 \pmod{20}$.

i) What is the remainder of dividing b by 20?

ii) What is the remainder of dividing b^2 by 20?

c) Let $n > 7$. Prove that $n! + 5$ is not a prime number.

QUESTION 5. (10 points) a) Convert $(443)_5$ to its equivalent number in base 10.

b) Convert $(221)_3$ to its equivalent number in base 7.

c) Find $(337)_8 \times (77)_8$. Give your answer in base 8.

QUESTION 6. (12 points) Let G be the simple graph with vertices $V = \{1, 4, 6, 9, 11, 14, 16, 19, 21, 24\}$. Two vertices v_1, v_2 are connected by an edge if 5 is a factor of $v_1 + v_2$ (i.e. if 5 divides $(v_1 + v_2)$). Show that G is $K_{5,5}$. What is the diameter of G ? What is the girth of G ?

QUESTION 7. (8 points) Let G be the simple graph with vertices $V = \{1, 2, 5, 7, 8\}$. Two vertices v_1, v_2 are connected by an edge if 3 is a factor of $(v_1 + v_2)$. Show that G is a planar.

QUESTION 8. (6 points) Draw two graphs G and \overline{G} such that G has exactly 4 vertices, \overline{G} is a planar, and one of the vertices in \overline{G} has degree equals 3.

QUESTION 9. (10 points) Use math induction to prove that for each $n \geq 1$ 3 is a factor of $n^3 - n$

QUESTION 10. (5 points) Let $\{a_n\}$ be a sequence such that a_n is given as a linear recurrence. If $\text{char}(a_n) = (\alpha - 4)^2(\alpha - 3)^3$. Find a general mathematical equation that describes a_n .

QUESTION 11. (10 points) Given $a_0 = 1$, $a_1 = 9$ and $a_n = 6a_{n-1} - 9a_{n-2}$. Find a mathematical equation that describes a_n . Find the 100th term of a_n (Do not simplify your answer)

EXTRA CREDIT (8 points) Let $a, b \in \mathbb{Z}^+$ such that 7 is a factor of $a^2 + b^2$. Prove that 7 is a factor of a and 7 is a factor of b . (USE THE BACK)