

HW number EIGHT, MTH 320, SPRING 2009

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QUESTION 1. Consider the group $(Z, +)$, and let H be a subgroup of Z . Show that $H = nZ$ for some $n \geq 1$.

QUESTION 2. a)Let $M = \{2, 4, 6, 8, 10, 12\}$. Show that $(M, \times_{14})$ is a cyclic group.

b) Let $H = \{6, 8\}$. Show that $(H, \times_{14})$ is a normal subgroup of M (M as in part (a)).

c) Now consider the quotient group $(M/H, \wedge)$. Find all elements of of M/H. Find the order of each element of M/H. Is $(M/H, \wedge)$ a cyclic group? explain.

QUESTION 3. Let $(M, *)$ be a finite abelian group of order n. Let $a_1, a_2, ..., a_n$ be the elements of M.
a) If n is an odd number, then show that $a_1 * a_2 * a_3 * ... * a_n = e$.

b) If n is an even number, then show that $x = a_1 * a_2 * a_3 * ... * a_n \in M$ and x is of order 2.

c) (From the book) Consider the numbers 1, 2, 3, ..., 10: can you add + or - for each number so that when you add them, then you get zero? any ideas? can you generalize your conclusion? can you prove it?

d) (From the book) consider the numbers 1, 2, 3, ..., 19: can you add + or - for each number so that when you add them, then you get zero? any ideas? can you generalize your conclusion? can you prove it?

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