HW number three, MTH 320, SPRING 2009

Ayman Badawi

QUESTION 1. (From the book) Let $S = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in R \setminus \{0\} \right\}$, and let * be the normal multiplication of matrices. Show that

(S, *) is a group. (You need to show closure, you need to find an identity, you need to show that for each $c \in S$, there is a $c^{-1} \in S$, YOU DO NOT NEED TO SHOW THE ASSOCIATIVE since we know it is true for matrices under normal multiplications. (This problem is indeed WAWWW NICE)

QUESTION 2. a set (S,*) is called a left-cancelative set if whenever a, b, c are elements in S (not necessary distinct) such that $a * b = a * c$, then b = c. Also, a set (S,*) is called a right-cancelative set if whenever a, b, c are elements in S (not necessary distinct) such that $b * a = c * a$, then b = c.

a)Prove that every group (M, *) is both left-cancelative and right-cancelative.

b) Give me an EXAMPLE of a monoid (M, *) such that (M, *) is neither left-cancelative nor right-cancelative.

QUESTION 3. a) Let $(M, *)$ be group and (H, *) be a subgroup of (M, *). Suppose there is an $a \in M \setminus H$ and choose an element $h \in H$. Prove that the left coset $a * H$ is the same as the left coset $a * h * H$.

b)Let $(M, *)$ be group and (H, *) be a subgroup of (M, *). Suppose there is an $a \in M \setminus H$ and suppose that $a * H = b * H$ for some $b \in M$. Show that $b \in a * H$.

QUESTION 4. Let $(M, *)$ is a group. Then

a) Suppose that $a * b = b * a$ for some $a, b \in M$. Prove that $a^{-1} * b^{-1} = b^{-1} * a^{-1}$ and $a * b^{-1} = b^{-1} * a$.

b) Suppose that $a \in M$ and $ord(a) =\mid a \mid= m$. Show that $ord(a^{-1}) = m$.

c) Let $\alpha = (234)o(234)o(13425) \in S_5$. Find $ord(\alpha)$.

QUESTION 5. a)Let $(M, *)$ be a finite set with a binary operation $*$ on M. Assume that (M, *) satisfies : Closure property, Associative Property, and assume that (M, *) is left-cancelative and right-cancelative. Prove that (M, *) is a group.

b) Give me an example of a semigroup (M, *) that satisfies the cancelation property but it is not a group.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`