

# MTH 101 (Final Sample)

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1. A survey of 850 family houses showed that 220 own Laptops, 450 own PCs, and 130 own both Laptops and PCs.
    - a. How many houses in the survey own either a Laptop or a PC?  
(a) 280    (b) 540    (c) 800    (d) 720    (e) None of the above
    - b. How many own a Laptop and do not own a PC?  
(a) 220    (b) 320    (c) 90    (d) 350    (e) None of the above
    - c. How many do not own any of the items: Laptops or PCs?  
(a) 50    (b) 80    (c) 310    (d) 350    (e) None of the above
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2. If  $A$  and  $B$  are events in a sample space set  $S$  and  $P(A) = 0.5$ ,  $P(B) = 0.3$ , and  $P(A \cap B) = 0.1$ . Then
    - a.  $P(A \cap B')$  =  
(a) 0.2    (b) 0.3    (c) 0.1    (d) 0.4    (e) None of the above
  3.  $P((A \cup B)')$  =  
(a) 0.3    (b) 0.2    (c) 0.1    (d) 0.4    (e) None of the above
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4. In a family with 3 children, excluding multiple births, what is the probability of having 2 girls and 1 boy in any order  
(a)  $\frac{3}{9}$     (b)  $\frac{2}{3}$     (c)  $\frac{1}{3}$     (d)  $\frac{3}{8}$     (e) None of the above
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5. A committee of 8 people must be selected from a pool of 7 women and 12 men. What is the probability that the committee has
  - a. 5 women and 3 men?
  - b. At least 1 woman?
  - c. All women?
6. By examining the past driving records of drivers in a certain city, an insurance company has determined the following empirical probabilities.

### Miles driven per year

	Less than 10,000 (M1)	10,000-15,000 (M2)	More than 15,000 (M3)	Total
Accident A	0.03	0.07	0.1	0.2
No Accident A'	0.12	0.33	0.35	0.8
Total	0.15	0.40	0.45	1

- If a driver is selected at random, what is the probability that the driver
- a. drives less than 10,000 miles or has no accident
  - b. drives 10,000 or more miles per year and has an accident
7. A fair die is painted so that 2 sides are red, 3 sides are white, and 1 side is blue. If the die is rolled once, what is the probability that the top side is white?
  8. An opinion poll is to be conducted among college students. 9 multiple choice questions each with 4 possible answers, will be asked. In how many ways can a student complete the poll, if exactly one response is given to each question?
  9. How many four-digit numbers can be formed from the numbers 1, 2, 3,4,5, 6 if the first letter must be 3?
  10. How many five-letter words start with the letter  $a$  and end with the letter  $m$ ?
  11. A product can be sold for \$29 per unit. The cost  $c$  of producing  $x$  units is  $c(x) = 24x + 5000$ .
    - a. What is the revenue function?
    - b. How many units must be sold for the company to break even?
    - c. How many units must be sold for the company to have \$300 profit?
  12. The management of a company that manufactured tables has fixed cost of \$275 per day and

a total cost of \$4500 per day at a daily output of 75 tables.

- a. Assuming the total cost per day,  $C(x)$ , is linearly related to the total output  $x$  per day. Write an equation for the total cost.
  - b. What is the total cost for an output of 32 tables?
  - c. Graph the cost function.
13. XYZ Utilities uses the following rates to compute the monthly cost of natural gas for residential customers.
- \$0.7675 per CCF for the first 50 CCF**
- \$0.6400 per CCF for the next 150 CCF**
- \$0.6130 per CCF for all over 200 CCF**
- a. Write a piecewise function  $C(x)$  for the cost of consuming  $x$  CCF (cubic hundred feet) of natural gas.
  - b. How much does it cost for a family consumed 215 CCF.
  - c. Graph the function  $C(x)$ .
14. A small resort must add a swimming pool to compete with a new resort built nearby. The pool will cost \$28,000. The resort borrows the money and agrees to repay it with equal payments at the end of each quarter for  $6\frac{1}{2}$  years at interest rate of 6%, compounded quarterly. Find the amount of each payment.
15. A firm of attorneys deposits \$500 of profit sharing money at the end of each semiannual period for  $7\frac{1}{2}$  years with an interest rate of 5%, compounded semiannually.
- a. Find the final amount in the account.
  - b. Find the amount of interest earned.
16. A small business borrows \$50,000 for expansion at 12% compounded monthly. The loan is due in 4 years
- a. Find the value of the loan after 4 years.
  - b. How much interest will the business pay?
17. In order to save enough money for the down payment on a condominium, a young couple deposits \$200 each month into an account that pays 7.02% interest compounded monthly. If they need \$10,000 for a down payment.
- a. How many deposits will they have to make?
  - b. How much interest will be paid?
18. The manager of a large apartment complex has found that the profit is given by the following function  $p(x) = -x^2 + 250x - 15000$ , where  $x$  is the number of units rented.
- a. For what value of  $x$  does the complex produce the maximum profit?
  - b. What is the maximum profit?
  - c. Graph the profit function.
19. Suppose that the revenue and cost functions for a small business are
- $$R(x) = x(70 - x), \text{ and}$$
- $$C(x) = 20x, \text{ where } 0 \leq x \leq 70.$$
- a. Find the break-even point(s).
  - b. Find the maximum revenue.
  - c. Sketch the graph of  $R(x)$  and  $C(x)$  on the same coordinates showing the intercepts and the vertex (for  $R(x)$ ). Indicate the regions of loss and profit.
20. How long will it take 10,000 to double if it is invested at 6.5% compounded monthly?
21. Suppose \$600 is deposited each quarter into an account paying 8% compounded quarterly.
- a. Find the value of the account after 4 years.
  - b. Find the value of the account after 3 years.
  - c. Find the interest earned during the fourth year.
22. If you paid \$45 to a loan company for the use of \$1200 for 90 days, what simple annual rate of interest did they charge?
23. Suppose \$5000 is invested into account for 18 months. Find the value of the account after the 18 months if the interest rate is 6%
- a. Simple interest

- b. Compounded annually
  - c. Compounded continuously
24. At a price of \$2 per unit of a certain product, the demand is 21 thousand units and when the price rises to \$5 per unit the demand decreases to 12 thousand units.
- a. Assuming the price-demand relationship is linear, find an equation for this relationship.
  - b. If the price-supply equation is given by  $p = 0.208x + 2.5$  where  $x$  is the number of items produced in thousand units, find the equilibrium point.
  - c. Graph the price-demand and price-supply equations in the same coordinate system. Clearly mark the equilibrium point on the graph.
25. Lina borrows \$7000 at 8% interest rate compounded monthly and plans to amortize the loan over 5 years in equal monthly payments:
- a. What is Lina's monthly payment?
  - b. How much interest will Lina pay?
26. At most 60 pounds of chocolates and 100 pounds of mints are available to make up 5-pound boxes of candy. A regular box has 4 pounds of chocolates and 1 pound of mints and sells for \$10. A deluxe box has two pounds of chocolates and 3 pounds of mints and sells for \$16. How many boxes of each kind should be made to maximize the revenue?
27. Use Gauss-Jordan elimination to solve

$$\begin{aligned} 3x + 4y + 6z &= 3 \\ x + y + 2z &= 1 \\ -2x + 3y - 3z &= -1 \end{aligned}$$

28. A corporation wants to lease a fleet 24 cars with combined capacity of 420 passengers. The three available types of cars carry 15, 20 and 25 passengers, respectively. How many of each type of car should be leased? Give at least three solutions.
29. Find the maximum and minimum for

$$\begin{aligned} z &= 2x + 3y \\ \text{subject to :} \\ x + 2y &\leq 8 \\ -x + 2y &\geq 0 \\ x &\geq 2 \\ x, y &\geq 0 \end{aligned}$$

30. A builder makes three kinds of houses. Type A requires 1 acre of land, \$60,000 in costs and 400 labor hours and returns a profit of \$20,000. Type B requires 1 acre of land, \$60,000 in costs and 300 labor hours, and returns a profit of \$18,000. Type C requires 2 acres of land, \$80,000 in costs and 300 labor hours and returns a profit of \$24,000. The builder has available a maximum of 60 acres of land, \$3,200,000 of capital and 18,000 labor hours. Find the number of houses of each type that should be built in order to maximize the profit.

Write the standard linear programming problem and do not attempt to solve it.

31. Use the simplex method to solve the following dual linear programming problem.

$$\begin{aligned} \text{Min } C &= 3x_1 + 5x_2 \\ \text{Subject to :} \\ x_1 + x_2 &\geq 6 \\ x_1 + 2x_2 &\geq 20 \\ -2x_1 + x_2 &\geq -8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

32. Use simplex method to solve the following linear programming problem

$$\text{Maximize } Z = 4x_1 + 2x_2 - 14x_3$$

Subject to :

$$3x_1 + 3x_2 - 6x_3 \leq 51$$

$$5x_1 + 10x_3 \leq 99$$

$$x_1, x_2, x_3 \geq 0$$

**33.**

$$\text{Maximize } Z = 4x + 5y$$

Subject to :

$$10x - 5y \leq 100$$

$$20x + 10y \geq 150$$

$$x, y \geq 0$$

- 34.** If 20 pounds of rice and 10 pounds of potatoes cost 16.20 and 30 pounds of rice and 12 pounds of potatoes cost 23.04, how much will 10 pounds of rice and 50 pounds of potatoes cost?
- 35.** A furniture company makes tables and chairs. The total number of tables and chairs produced must be at least 60 per week. Sales experience has shown that at least 1 table must be made for every 3 chairs that are made. If it costs \$152 to make a table and \$40 to make a chair, how many of each should be produced each week to minimize the cost?
- 36.** The final augmented matrix of a system of linear equations problems is given as follows. For each system find one solution, if any.

$$\text{a. } \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right] \quad \text{b. } \left[ \begin{array}{ccc|c} 1 & 0 & -7 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \text{c. } \left[ \begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 5 \end{array} \right]$$

- 37.** Use geometric (graphing) method to solve

$$\text{Maximize and Minimize } Z = -4x_1 + x_2$$

subject to :

$$2x_1 + 3x_2 \geq 12$$

$$x_1 - 3x_2 \leq 3$$

$$x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

38. (10 Points) Use the simplex method to solve

$$\text{Max } P = 3x_1 - 2x_2 + 4x_3$$

subject to :

$$2x_1 - 3x_2 + 3x_3 \leq 45$$

$$4x_1 - x_2 + 2x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

39. Form the dual problem (**Do not solve**) of the following minimization problem

$$\text{Minimize } C = 4x_1 + 3x_2$$

Subject to:

$$2x_1 + x_2 \geq 9$$

$$x_1 - 4x_2 \geq 3$$

$$-8x_1 + 5x_2 \geq -20$$

$$x_1, x_2 \geq 0.$$