Sample Problems (Exam 2)

1. At most 60 pounds of chocolates and 100 pounds of mints are available to make up 5-pound boxes of candy. A regular box has 4 pounds of chocolates and 1 pound of mints and sells for \$10. A deluxe box has two pounds of chocolates and 3 pounds of mints and sells for \$16. How many boxes of each kind should be made to maximize the revenue?

2. Use Gauss-Jordan elimination to solve

$$\begin{aligned} 3x + 4y + 6z &= 3 \\ x + y + 2z &= 1 \\ -2x + 3y - 3z &= -1 \end{aligned}$$

3. A corporation wants to lease a fleet 24 cars with combined capacity of 420 passengers. The three available types of cars carry 15, 20 and 25 passengers, respectively. How many of each type of car should be leased? Give at least three solutions.

4. Use Gauss-Jordan elimination to solve

$$\begin{aligned} 2x + y + 5z &= 600 \\ x - 2y &= 0 \\ x + \quad y + \quad z &= 200 \end{aligned}$$

5. Find the maximum and minimum for

$$\begin{aligned} z &= 2x + 3y \\ \text{subject to} &: \\ x + 2y &\leq 8 \\ -x + 2y &\geq 0 \\ x &\geq 2 \\ x,\ y &\geq 0 \end{aligned}$$

6. A company that rents small moving trucks wants to purchase 25 trucks with a combined capacity of 28000 cubic feet. Three different types of trucks are available: a 10-foot truck with a capacity of 350 cubic feet, a 14-foot truck with a capacity of 700 cubic feet, and a 24-feet truck with a capacity of 1,400 cubic feet. How many of each type of truck should the company purchase? Give at least two solutions. A builder makes three kinds of houses. Type A requires 1 acre of land, \$60,000 in costs and 400

labor hours and returns a profit of $20,000. Type B requires 1 acre of land, $60,000 in costs and 300 labor hours, and returns a profit of $18,000. Type C requires 2 acres of land, $80,000 in costs and 300 labor hours and returns a profit of $24,000. The builder has available a maximum of 60 acres of land, $3,200,000 of capital and 18,000 labor hours. Find the number of houses of each type that should be built in order to maximize the profit. Write the standard linear programming problem and do not attempt to solve it.

7. Use the simplex method to solve the following dual linear programming problem.

$$\begin{aligned} \text{Min } C &= 3x_1 + 5x_2 \\ \text{Subject to } &: \\ x_1 + x_2 &\geq 6 \\ x_1 + 2x_2 &\geq 20 \\ -2x_1 + x_2 &\geq -8 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

Use simplex method to solve the following linear programming problem

$$\begin{aligned} \text{Maximize } Z &= 4x_1 + 2x_2 - 14x_3 \\ \text{Subject to } &: \\ 3x_1 + 3x_2 - 6x_3 &\leq 51 \\ 5x_1 + 10x_3 &\leq 99 \\ x_1,\, x_2,\, x_3 &\geq 0 \end{aligned}$$

8. A shop sells skirts for $45 and blouses for $35. Its entire stock is worth $51750. But sales are slow and only half the skirts and two thirds of the blouses are sold for a total of $30600. How many skirts and blouses are left in the store?

9.

$$\begin{aligned} \text{Maximize } Z &= 4x + 5y \\ \text{Subject to } &: \\ 10x - 5y &\leq 100 \\ 20x + 10y &\geq 150 \\ x,\, y &\geq 0 \end{aligned}$$

10. If 20 pounds of rice and 10 pounds of potatoes cost 16.20 and 30 pounds of rice and 12 pounds of potatoes cost 23.04, how much will 10 pounds of rice and 50 pounds of potatoes cost?

11. A furniture company makes tables and chairs. The total number of tables and chairs produced must be at least 60 per week. Sales experience has shown that at least 1 table must be made for every 3 chairs that are made. If it costs \$152 to make a table and \$40 to make a chair, how many of each should be produced each week to minimize the cost?

12. The final augmented matrix of a system of linear equations problems is given as follows. For each system find one solution, if any.

a. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 0 & -7 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ c. $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 5 \end{array}\right]$

13. Use geometric (graphing) method to solve

$$\begin{aligned} \text{Maximize and Minimize } Z &= -4x_1 + x_2 \\ \text{subject to } &: \\ 2x_1 + 3x_2 &\geq 12 \\ x_1 - 3x_2 &\leq 3 \\ x_2 &\leq 4 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

14. (10 Points) Use the simplex method to solve

$$\begin{aligned} \text{Max } P &= 3x_1 - 2x_2 + 4x_3 \\ \text{subject to } &: \\ 2x_1 - 3x_2 + 3x_3 &\leq 45 \\ 4x_1 - x_2 + 2x_3 &\leq 20 \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

15. Form the dual problem **(Do not solve)** of the following minimization problem

$$\begin{gathered} \text{Minimize} \quad C = 4x_1 + 3x_2 \\ \text{Subject to:} \\ 2x_1 + x_2 \geq 9 \\ x_1 - 4x_2 \geq 3 \\ -8x_1 + 5x_2 \geq -20 \\ x_1, x_2 \geq 0. \end{gathered}$$