$\left(\dfrac{4}{4}\right)$ **1.** A group of students is planning to rent a bus and organize a one day trip to Abu Dhabi. They will rent one bus and each student will have to pay for the open buffet where they plan to eat. If 12 students participate in the activity, the total costs (one Bus and Restaurant for all) will be 2400 Dhs. If 20 students participate, the total costs will be 3000 Dhs. Write a linear function $C(x)$ for the total costs of having x students participating in the trip.

$$C(x) = a + bx \quad , \quad \text{where } a = \text{fixed costs}$$
$$b = \text{variable cost per unit.}$$

$$a + b(12) = 2400 \longrightarrow (1)$$
$$a + b(20) = 3000 \longrightarrow (2)$$

solving simultaneously :

$$\cancel{a} + 12b = 2400$$
$$-\ \cancel{a} + 20b = 3000$$
$$\rule{2cm}{0.4pt}$$
$$\cancel{+}\ 8b = \cancel{+}\ 600$$
$$b = 75$$

or

Calculate the slope directly:
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3000 - 2400}{20 - 12} = 75$$

So $b = 75 = $ slope

(the slope is the coefficient of x)

substitute $b = 75$ in (1) :

$$a + (75 \times 12) = 2400$$
$$a = 1500$$

$$\therefore \quad C(x) = 1500 + 75x \quad \checkmark$$

$\left(\dfrac{4}{4} \right)$

2. A company, offering Refund Anticipation Loans (RAL), has a customer who wants to borrow a RAL Amount of $600 for 21 days. At the end of 21 days, he will have to reimburse the RAL Amount plus some RAL Fees calculated according to the following table

RAL Amount	RAL Fee
$100 – $500	29
$501 – $1,000	39
$1,001 – $5,000	109
$5,001 – $10,000	229

What annual simple interest rate is he charged?

At a RAL amt. of $600, RAL Fee = 39

$$I = Prt$$

- $I = 39$
- $P = 600$
- $r = ?$
- $t = \dfrac{21}{360}$

$$39 = 600 \times r \times \dfrac{21}{360}$$

$$\therefore r = 111.43\%. \quad \checkmark$$

(NB: we assume 1 yr = 360 days) $\checkmark$

$$\left(\frac{8}{8}\right)$$

3. A company manufactures snow-boards. In order to analyze their business, they collected data regarding Price-Demand and Costs from which its Marketing department, using statistical analysis, determined the following functions:

Price-Demand ($)	$p(x) = 500 - 10x$
Costs (thousands $)	$C(x) = 1500 + 50x$
Revenue (thousands $)	$R(x) = 500x - 10x^2$
Profit (thousands $)	$P(x) = 450x - 10x^2 - 1500$

where x is in thousands of snow-boards.
  a. Calculate the *break-even* points to the nearest hundred snow-boards
  b. For what output x is the **Revenue** Maximum?
  c. What is the maximum **Revenue**?
  d. At what price is the **Revenue** maximum?

a) At the break-even points, $P(x) = 0$

$$0 = 450x - 10x^2 - 1500$$

$$10x^2 - 450x + 1500 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad , \text{ where } \begin{array}{l} a = 10 \\ b = -450 \\ c = 1500 \end{array}$$

$$= \frac{-(-450) \pm \sqrt{(-450)^2 - 4 \times 10 \times 1500}}{2 \times 10}$$

$$= \frac{450 \pm \sqrt{142\,500}}{20}$$

$$= 41.375 \quad \text{or} \quad 3.625$$

∴ Break-even points = 3600 units,   41 400 units. ✓

(to the nearest 100 units)

b) $R(x) = 500x - 10x^2$,   where $a = -10$

                                                    $b = 500$

$$h = \frac{-b}{2a} = \frac{-500}{2 \times -10} = 25$$

∴ Output at which revenue is maximum $= h$

$$= \underline{\underline{25 \text{ thousand units.}}} \checkmark$$

c)   Maximum revenue $= k = f(h) = R(25)$

∴ $R(25) = 500(25) - 10(25)^2$

$$= \underline{\underline{6250 \text{ thousands } \$}} \checkmark$$

d)   Price at which revenue is maximum $= p(h) = p(25)$

$$p(x) = 500 - 10x$$

∴ $p(25) = 500 - 10(25)$

$$= \underline{\underline{\$ 250 \text{ per unit}}} \checkmark$$

$\frac{4}{4}$

4. You want to make an investment. Which one is the best (A) 12% compounded semi-annually or (B) 12.4% compounded annually? Justify your answer.

- For option (A) :

$$APY = \left(1 + \frac{r}{m}\right)^m - 1$$

- $r = 0.12$
- $m = 2$

$$\therefore APY = \left(1 + \frac{0.12}{2}\right)^2 - 1$$

$$= 12.36\% \checkmark$$

- For option (B) ;

$$APY = 12.40\% \checkmark$$

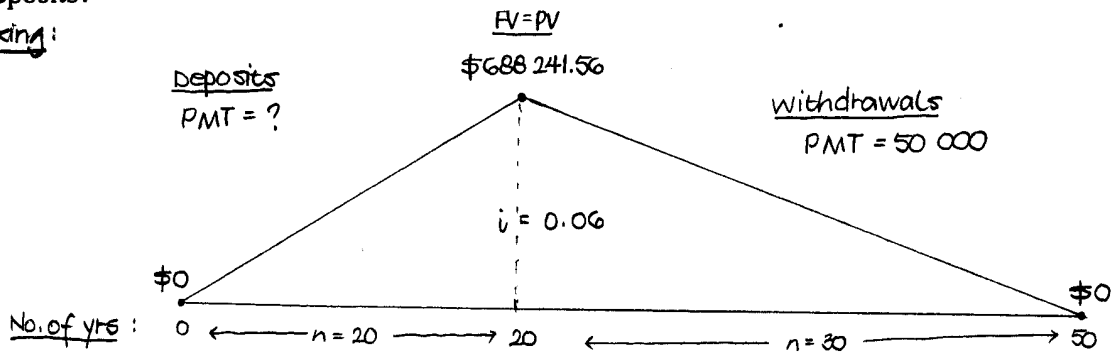$\therefore$ <u>Option (B) is the better investment.</u> $\checkmark$

It has a higher annual percentage yield than option (A). This means that you will get higher returns if you choose option (B), & it is therefore preferred.

5. Yassine wants to establish an annuity for retirement purposes. He is planning to make deposits at the end of each year for the next 20 years, and then, he wants to make withdrawal of $50,000 at the end of each year for 30 years. Assuming an interest rate of 6% compounded yearly, Yassine calculated that he needs to accumulate $688,241.56 in order to be able to make his withdrawals. What should be his annual deposits?

Working :

FV = PV
$688 241.56

Deposits
PMT = ?

withdrawals
PMT = 50 000

$i = 0.06$

$0

$0

No. of yrs :   0   ←—— $n = 20$ ——→ 20   ←———— $n = 30$ ————→ 50

$$FV = PMT \left[\frac{(1+i)^n - 1}{i}\right]$$

- FV = PV = 688 241.56
- PMT = ?
- $i = \dfrac{r}{m} = \dfrac{0.06}{1} = 0.06$
- $n = mt = 1 \times 20 = 20$

$$688\ 241.56 = PMT \left[\frac{(1+0.06)^{20} - 1}{0.06}\right]$$

$$688\ 241.56 = PMT\ (36.79)$$

$$\therefore PMT = \$18\ 709.54 \quad \checkmark$$

6. Sakina also established an annuity for retirement purposes several years ago. She has a guaranteed interest rate of 8% per year. Over many years of deposits, she managed to accumulate $407,909.89 into an account. She calculated that, with such an amount, she has just enough money to make withdraws of $35,000 at the end of every year for 35 years, at which time the account will have a zero balance. She rigorously followed that plan and now 30 years are gone (and 30 withdrawals have been made).

  a. How much money is in Sakina's account? ·

  [Hint, at all time, there is just enough money in Sakina's account to be able to make the remaining withdrawals]

  b. How much interest did Sakina earn during this 30 year period?

Data from question :

$$PV = 407\ 909.89 \quad , \quad PMT = 35\ 000 \quad , \quad i = \frac{0.08}{1} \quad , \quad n = 35 \times 1$$

a) Money in account after 30 yrs = PV of $(35-30)$ yrs

$$PV = PMT \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

- $PV = ?$
- $PMT = 35\ 000$
- $i = 0.08$
- $n = 35 - 30 = 5$

$$\therefore PV = 35\ 000 \left[\frac{1 - (1+0.08)^{-5}}{0.08} \right]$$

$$= 35\ 000\ (3.99)$$

$$= \$ 139\ 744.85 \quad \checkmark$$

$\longrightarrow$

b)

$$A = P + C + I$$

- A = amt. at end of period
  = 139 744.85

- P = amt. at beginning of period
  = 407 909.89

- C = net contributions over the period
  = $-35\,000 \times 1 \times 30$ = $-1\,050\,000$

- I = interest earned over the period
  = ?

$$139\,744.85 = 407\,909.89 - 1\,050\,000 + I$$

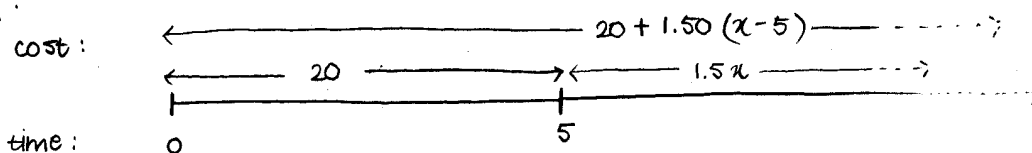$$\therefore I = \$781\,834.96 \checkmark$$

$\left(\dfrac{4}{4}\right)$

7. *Telephonics*, a phone company, has the following costs for long distances to Montreal

First 5 minutes: 20 Dhs
or less
Over 5 minutes 1.5 Dhs per minutes

Write a piecewise defined function for the cost $T(x)$ of a phone call of x minutes.

Working :

cost :

$$\longleftarrow \qquad\qquad 20 + 1.50\,(x-5) \longrightarrow\cdots\cdots\cdots\rangle$$

$$\longleftarrow \quad 20 \quad \longrightarrow\langle\!\longleftarrow 1.5\,x \longrightarrow\cdots\cdots\rangle$$

$$\vdash\!\!\longrightarrow\qquad\qquad\qquad\vdash\qquad\qquad\qquad\longrightarrow\rangle$$

time : $\quad$ o $\qquad\qquad\qquad$ 5

$$T(x) = \begin{cases} 20.00 & 0 \leqslant x \leqslant 5 \\[2em] 20.00 + 1.50\,(x-5) & 5 < x \end{cases}$$

when simplified :

$$T(x) = \begin{cases} 20 & 0 \leqslant x \leqslant 5 \\[2em] 1.5\,x + 12.5 & 5 < x \end{cases} \quad \checkmark$$