8

1. (8 points) A company manufactures memory chips. Its marketing research department collected data about the price-demand relation and the cost function. An analyst produced the following price-demand and cost functions to model the data:

price-demand	$p(x) = 75 - 5x$	$1 \le x \le 15$
cost	$C(x) = 150 + 10x$	$1 \le x \le 15$

where (1) p is the wholesale price per memory chip at which x million memory chips can be sold and (2) $C(x)$ is in millions of dollars.

a. Find the Revenue function $R(x)$ and the Profit function $P(x)$

$R(x) = x\,p(x) = x(75-5x) = 75x - 5x^2$ $\quad \boxed{\therefore R(x) = 75x - 5x^2}$

$P(x) = R(x) - C(x) = 75x - 5x^2 - [150 + 10x] = 75x - 5x^2 - 150 - 10x$

$\boxed{P(x) = 65x - 5x^2 - 150}$

$a = -5$
$b = 65$
$c = -150$

b. Find the break even points

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-65 \pm \sqrt{65^2 - 4 \cdot -5 \cdot -150}}{2 \cdot -5} = 3$ and 10

$\boxed{\therefore \text{BEP at output of 3 million and 10 million}}$

c. Find the vertex of $P(x)$ and indicate (1) the production level which maximizes the profit and (2) the maximum profit.

$h = \dfrac{-b}{2a} = \dfrac{-65}{2 \cdot -5} = 6.5$ $\quad k = 65(6.5) - 5(6.5^2) - 150 = 61.25$

vertex $= (6.5, 61.25)$

$\boxed{\text{(2) Max profit is \$61.25 million}}$ at output of $\boxed{\text{(1) 6.5 million}}$

\$61 250 000 $\qquad$ 6 500 000 units

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2. (5 points) The manager of a small boutique has a special technique to establish the price at which he sells his items: if he pays $80 for an item, he will sell it for $142; if he pays $50 he will sell it for $92. Assuming that the price p at which he sells the items is linearly related to the cost x at which he pays them, find an equation that relates p and x. Write your answer in the form $p = mx + b$.

$x \quad p$

$(80, 142)$

$(50, 92)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{142 - 92}{80 - 50} = \frac{5\cancel{0}}{3\cancel{0}} = \frac{5}{3} \quad \checkmark$$

$$92 = \frac{5}{3}(50) + b$$

$$92 - \frac{5}{3}(50) = b = \frac{26}{3} \quad \checkmark$$

$$\therefore y = \frac{5}{3}x + \frac{26}{3}$$

$$\boxed{\therefore p = \frac{5}{3}x + \frac{26}{3}} \quad \checkmark$$

8

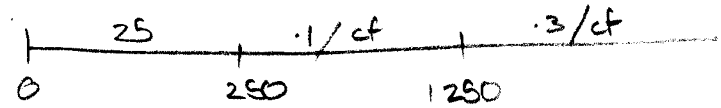
3. (8 points) A state owned company has noticed that, in spite of prohibition to water the garden during the drought season, people continue to do so which negatively impact the capacity of the company to adequately supply water. They have therefore decided to produce a price structure designed to discourage people from using an excessive quantity of water during the summer. The monthly costs for water during the summer is given by the following table

$25 for the first 250 cubic ft or less

$0.1 per cubic ft for the next 1000 cubic ft

$0.3 per cubic ft for all over 1250 cubic ft

a. Write a piecewise function $C(x)$ for the cost of using x cubic ft of water in a summer month?



$$C(x) = \begin{cases} 25 & \text{if } 0 \le x \le 250 \\ 25 + 0.1(x-250) & \text{if } 250 < x \le 1250 \\ 25 + 0.1(1000) + 0.3(x-1250) & \text{if } x > 1250 \end{cases}$$

✓

b. Calculate $C(1000)$

$C(1000) = 25 + 0.1(1000 - 250) = \$100$

✓

6

Excellent!

4. (6 points) Sakina pays $5,000 for 90-day note which yields 8% simple interest (both interest and principal will be paid at the end of 90 days). Wishing to be able to use her money sooner, Sakina sells the note to Yassine for $5,090 after 30 days. [In this problem a 360-day year is assumed]

a. What annual *simple interest* rate will Sakina receive for her investment?

$A = P(1+rt)$ $A = 5090$ $P = 5000$ $t = \frac{30}{360}$

$\frac{A}{P} = 1 + rt$

$\frac{\frac{A}{P} - 1}{t} = r = \frac{\frac{5090}{5000} - 1}{\frac{30}{360}} = .216$

Sakina's $r = .216 = \boxed{21.6\ \%}$

b. What annual *simple interest* rate will Yassine receive for his investment?

1 before: $A = P(1+rt)$ $P = 5000$ $r = .08$ $t = \frac{90}{360}$ $A = 5000\left(1 + .08 \cdot \frac{90}{360}\right)$

$A = \$5100$

2 resell: $A = 5100$ $P = 5090$ $r = ?$ $t = \frac{60}{360}$

$A = P(1+rt)$

$\frac{A}{P} = 1 + rt$

$\frac{\frac{A}{P} - 1}{t} = r = \frac{\frac{5100}{5090} - 1}{\frac{60}{360}} = .0117878$

$\therefore$ Yassine's $r = .0117878 = \boxed{1.179\ \%}$

4

5. (4 points) You need to invest some money. Bank ABC offers guaranteed investments at a rate of 5% compounded weekly. Bank XYZ offers guaranteed investments at a rate of 5.2% compounded semi-annually. Which bank would you choose and why?

$\boxed{ABC}$

$APY = \left(1+\frac{r}{m}\right)^m - 1 \qquad r = .05 \qquad m = 52$

$APY = \left(1+\frac{.05}{52}\right)^{52} - 1 = .051246$

$\boxed{XYZ}$

$r = .052 \qquad m = 2$

$APY = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{.052}{2}\right)^2 - 1 = .052676$

∴ I'll choose Bank XYZ because their annual percentage yield (5.27%) is higher than Bank ABCs (5.12%) so I will earn more interest with XYZ

8

6. (8 points) Wishing to plan for retirement, a worker makes annual deposits of $1,000 into a bank account earning 8% compounded annually. He makes his first deposit on his 36^{th} birthday and his last deposit on his 60^{th} birthday (25 equal deposits in all). Starting at the age of 60, the worker plans to make 15 annual withdrawals at the end of each year.

a. How much money will be in the account after 25 years of deposits?

$PMT = 1000 \quad r = .08 \quad i = \frac{.08}{1} \quad n = \frac{25}{1}$
$m = 1$

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right) = 1000\left(\frac{1.08^{25} - 1}{.08}\right) = \boxed{\$73\,105.94}$$

b. Find the amount of each withdrawal

$PV = 73\,105.94 \quad r = .08 \quad i = \frac{.08}{1} \quad n = \frac{15}{1}$
$M = 1$

$$PMT = \frac{PV\,i}{1-(1+i)^{-n}} = \frac{73\,105.94 \cdot .08}{1 - 1.08^{-15}} = \boxed{\$8540.93}$$

c. How much interest will the worker earn during the whole 40-year period?

$A = 0 \quad P = 0 \quad C = \text{dep} - \text{with} = 1000 \cdot 25 - 8540.93 \cdot 15$

$C = -103\,113.95$

$I = A - P - C$
$= 0 - 0 - [-103\,113.95] = \boxed{\$103\,113.95}$

check

$I_{dep}: 73105.94 - 0 - 1000 \cdot 25 = \$48105.94$

$I_{wid}: 0 - 73105.94 + 8540.93 \cdot 15 = \$55\,008.01$

6

7. (6 points) A couple purchased a $120,000 home 10 years ago by paying 20% down and signing a 30-year mortgage at 12% compounded monthly.

a. What is the outstanding balance of the loan now that 10 years are gone?

PMT = ? $PV = 0.8 \times 120000 = 96000$ $r = .12$, $m = 12$ $i = \frac{.12}{12} = .01$ $n = 30.12 = 360$

$$PMT = \frac{PV\, i}{1-(1+i)^{-n}} = \frac{96000 \cdot .01}{1-(1.01)^{-360}} = \$987.47$$

10y later: $n = 20.12 = 240$ $PMT = 987.47$ $i = .01$

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right) = 987.47\left(\frac{1-1.01^{-240}}{.01}\right) = \boxed{\$89\,681.45}$$

b. How much interest did the couple pay during these 10 years?

$A = 89\,681.45$ $P = 96000$ $C = -987.47 \cdot 120 = -118\,496.40$

$$I = A - P - C$$
$$= 89\,681.45 - 96000 - [-118\,496.40] = \boxed{\$112\,177.85}$$

check $C = -118\,496.40$

reduction $= 96000 - 89\,681.45 = 6318.55$

$I + \text{red} = 112\,177.85 +$