Name Sana Valappil, ID g00093543

MTH 221, Exam I

Ayman Badawi

$\frac{43.5}{45}$

QUESTION 1. (i) Let A, B be 3×3 matrices such that $|A| = -8$ and $|B| = 4$. Then $|2A^T B^{-1}| =$

(a) -16 ✓ (b) -4 (c) 4 (d) -64

(ii) Let $A = \begin{bmatrix} 1 & 1 \\ 7 & 8 \end{bmatrix}$. Then $A^{-1} =$

(a) ✓ $\begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 7 \\ 1 & -8 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 \\ -7 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & -1 \\ -7 & -1 \end{bmatrix}$

(iii) Let $A = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -3 & 0 \\ -1 & -4 & -2 \end{bmatrix}$ Then $|A + B| =$

(a)6 (b) 12 ✓ (c) -6 (d) 0

(iv) One of the following is a subspace of R^3

(a)$\{(1,1,1), (-1,0,1)\}$ (b) ✓ $\{(a,\ b,\ a)|a + 3b = 0\}$ (c)$\{(a, ab, b)|a, b \in R\}$
(d) $\{(a,\ 2a + b,\ b)|a + b = 1\}$

(v) Let $D = \{(2a + 6b,\ a + 3b,\ -a - 3b)|a, b \in R\}$. One of the following is a basis for D

(a)$\{(2,1,1), (6,3,-3)\}$ (b) $\{(2,1,-1), (6,3,-3)\}$ (c) ✓ $\{(2,1,-1)\}$ (d) $\{(6,3,3)\}$

(vi) One of the following is a linear transformation

(a)$T : R^2 \to R$ such that $T(a,b) = 2a - b + 1$ (b) $T : R^3 \to R^2$ such that $T(a,b,c) = (ac, b)$
(c) ✓ $T : R^2 \to R^3$ such that $T(a,b) = (a - b, a + 2b, b)$ (d) $T : R^3 \to R^2$ such that $T(a,b,c) =$ ~~(ac, b)~~ $(0, abc)$

(vii) Let A be a 4×4 matrix such that $C_A(\alpha) = (\alpha - 1)^2(\alpha - 4)(\alpha - c)$. Assume that Trace($A$) = $|A|$. Then Trace(A) =

(a)5 (b) 7 (c) I need more information (d) 8 ✗ (e) ✓ 2

(viii) Let $T : R^2 \to R$ be a linear transformation such that $T(2,1) = 7$ and $T(0,1) = -3$. Then $T(6,5) =$

(a)18 (b) 5 (c) 6 (d) 15 ✓

note

Trace(A) = 1+1+4+c
|A| = 1×1×4×c
⇒ 6+c = 4c ⇒ c = 2

Hence
Trace(A) = 1+1+4+2
= 8

(ix) consider the system of linear equation

$$x + 3y + z = 10$$
$$-x - 2y + 23.67z = 108.896$$
$$-x - 3y + 5z = 32$$

. Use cramer rule. Then the value of z is

(a)1 (b) -1 (c) 6 (d) 7

-1.5

(x) Let $A = \begin{bmatrix} 1 & b & -6 \\ -1 & -2 & 12 \\ -1 & -b & c \end{bmatrix}$. Consider the system of linear equations

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ a \end{bmatrix}$$

. Then the system will have infinitely many solutions in one of the cases:

(a) $b = 77.98,\ c = 6$, and $a = 77.2$ (b) $b = 2,\ c = 6,\ and\ a = 98.678$

(c) $b = 2.3,\ c = 6.5,\ and\ a = 1$ (d) $b = 99.762,\ c = 6,\ and\ a = 1$

QUESTION 2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -1 \\ -2 & -2 & -1 \end{bmatrix}$. Find A^{-1} and $(A^T)^{-1}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ -2 & -2 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} R_1 + R_2 \to R_2 \\ 2R_1 + R_3 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$$-R_2 + R_1 \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \quad -R_3 + R_1 \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & -1 & -1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

5/5

QUESTION 3. Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$.

(i) Find all eigenvalues of A.

$$= (-1)^2 \alpha \begin{vmatrix} \alpha & 1 \\ -1 & \alpha - 2 \end{vmatrix} =$$

$$c_A(\alpha) = |I_3\alpha - A| = \begin{vmatrix} \alpha & 0 & 0 \\ -1 & \alpha & 1 \\ 0 & -1 & \alpha-2 \end{vmatrix} = \alpha(\alpha(\alpha-2)+1)$$

Chose 1st row

$$= \alpha(\alpha^2 - 2\alpha + 1)$$
$$= \alpha(\alpha-1)^2$$

$c_A(\alpha) = 0 \Rightarrow \alpha = 0, 1$ are the eigenvalues

4/4

(ii) For each eigenvalue α, find the eigenspace E_α

E_0:
$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \end{array}$$

$-x_2 - 2x_3 = 0$

$x_2 = -2x_3$

$-x_1 + x_3 = 0$

$x_1 = x_3$

$E_0 = \{(x_3, -2x_3, x_3) \mid x_3 \in \mathbb{R}\} = \text{span}\{(1,-2,1)\}$

E_1:
$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right] \end{array} \xrightarrow{R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right] \xrightarrow{R_2+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 0$

$x_2 + x_3 = 0$

$x_2 = -x_3$

$0 = 0$

$E_1 = \{(0, -x_3, x_3) \mid x_3 \in \mathbb{R}\} = \text{span}\{(0,-1,1)\}$

6/6

QUESTION 4. Let $T : R^4 \to R^3$ such that $T(a,b,c,d) = (b+c+d,\ -b-c-d,\ 2b+2c+2d)$

(i) Find the standard matrix presentation of T

$$M_T = \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 2 & 2 & 2 \end{bmatrix} \end{array} \qquad T = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

3/3

(ii) Find all points in the domain (i.e., in R^4) such that $T(a,b,c,d) = (2,-2,4)$

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 2 \\ 0 & -1 & -1 & -1 & -2 \\ 0 & 2 & 2 & 2 & 4 \end{array}\right] \xrightarrow[-2R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \left[\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

leading variable: b
free variables: a, c, d

$b+c+d = 2 \Rightarrow b = 2-c-d$

Soln. set $= \{(a, 2-c-d, c, d) \mid a, c, d \in \mathbb{R}\}$

(iii) Find a basis for the Range(T)

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 2 & 2 & 2 \end{bmatrix} \xrightarrow[-2R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

basis for Range(T) $= \{(1,-1,2)\}$

(iv) Find $Z(T) = Ker(T) = Null(T)$ and write it as span of independent points

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 2 & 2 & 2 & 0 \end{array}\right] \xrightarrow[\text{as (ii)}]{\text{same operations}} \left[\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$b = -c-d$

Soln. set $= \{(a, -c-d, c, d) \mid a, c, d \in \mathbb{R}\} = \text{span}\{(1,0,0,0), (0,-1,1,0), (0,-1,0,1)\}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu`, `www.ayman-badawi.com`