Department of Mathematics and Statistics
American University of Sharjah
Final Exam – Fall 2017
MTH 320 – Abstract Algebra I

Date: Saturday, December 16, 2017 **Time:** 8:00 am - 10:00 am

Student Name	Student ID Number

Instructor Name	Class Time
Dr. Ayman Badawi	M, W : 12:30-13:45

1. ***Do not open*** *this exam until you are told to begin.*
2. ***No*** *questions are allowed during the examination.*
3. *This exam has 2pages + this cover exam page.*
4. *Do not separate the pages of the exam.*
5. *Scientific calculators are allowed.*
6. *Turn off all cell phones and remove all headphones.*
7. *Take off your cap.*
8. *No communication of any kind is allowed during the examination*
9. *If you are found wearing a smart watch or holding a mobile phone at any point during the exam then it will be considered an academic violation and will be reported to the dean's office.*

Student signature: ________________________________

Name————————————, ID ——————

Final Exam: Abstract Algebra, MTH 320,Fall 2017

Ayman Badawi

Score = ———

QUESTION 1. (i) Let $D = Z_4 \times Z_3$. Then $H = \{0\} \times Z_3$ is a subgroup of D. Find all left cosets of H. The abelian group D/H is isomorphic to a well-known group F. What is F?

(ii) Give me an example of an infinite NON-ABELIAN group say D such that D has a simple normal subgroup H ($H \neq \{e\}$ and $H \neq D$) where D/H is abelien but not cyclic

(iii) Give me an example of a group say $(D, *)$ such that D has two elements a, b where $|a| = |b| = \infty$, but $|a * b| = 13$

QUESTION 2. Let D be an abelien group with $3^2 \cdot 13^2$ such that D has EXACTLY one subgroups with 13 elements. Up to isomorphism, find all possible structures of D.

QUESTION 3. Let D be an abelian group with $5 \cdot 7^2$ elements. Up to isomorphism, find all possible structures of D.

QUESTION 4.
Let $F : (Z, +) \to (\Re^*, .)$ be a nontrivial group homomorphism. Given $Ker(F) \neq \{0\}$. Find Range(F) and Ker(F). What is $F(12)$? What is $F(7)$?

QUESTION 5. Write $D = Z_4 \times Z_6 \times Z_{14}$ in terms of its invariant factors.

QUESTION 6. Assume H is a subgroup of $(D, *)$, where $H \neq D$. Assume that $D \setminus H$ is a finite set. Note that $D \setminus H = \{x \in D \mid x \notin H\}$. Prove that D is a finite group.

QUESTION 7. Consider the group (S_{15}, o). Prove that S_{15} has a subgroup with 56 elements. Let M be a cyclic a subgroup of S_{15} of maximal order, say m (i.e., the order of every cyclic subgroup of S_{15} is less or equal to m). Find m.

Faculty information

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