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Final Exam MTH 213 , Fall 2011

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QUESTION 1. (**WRITE DOWN T OR F. Each 1.5 points, total = 30 points**)

(i) If T is a partial order on a set A and $x, y \in A$, then either $x \bigwedge y = x$ or $x \bigwedge y = y$.

(ii) If $k \in \mathbb{N}$, then $\sqrt{k}$ is an irrational number.

(iii) If A is a set such that $|A| = 2$, then there are exactly 3 binary relation on A.

(iv) $-21(mod\ 11) = 10$.

(v) The number 1.81345 is a rational number.

(vi) the equation $4x \equiv 6(mod\ 10)$ has exactly two distinct solutions in Z_{10}.

(vii) There is no SOLUTION to the two equations $x \equiv 9(mod\ 12)$ and $x \equiv 3(mod\ 4)$

(viii) Let $F = \{a \in \mathbb{R} \mid a$ is irrational number $\}$. Then there is a bijection function from F onto [-4, 0].

(ix) $K_{10,2}$ is a planar.

(x) $TG(Z_{125})$ is a connected graph.

(xi) If $a, b, d \in \mathbb{N}$ and $d = na + mb$ for some $n, m \in \mathbb{Z}$, then $gcd(a, b) = d$.

(xii) If T is a tree with n vertices, then $|E| = (n(n-1))/2$

(xiii) It is possible to have a planar with 22 edges and 9 vertices.

(xiv) It is possible to have an Euler Graph with 13 vertices and 12 edges.

(xv) It is possible to have a Hamilton Graph with 13 vertices and 12 edges.

(xvi) $K_{10,1}$ is a tree

(xvii) If T is a partial order on $\mathbb{N}$ and $(13, 4) \in T$, then $4 \bigwedge 13 = 13$.

(xviii) If T is a total order on a set A and $b \bigvee a = a (a \neq b)$, then $(b, a) \in T$ but $(a, b) \notin T$

(xix) $TG(Z_{16})$ has exactly two components such that each component is complete.

(xx) $K_{3,3}$ has a subgraph H where H is a planar.

QUESTION 2. (**Circle the correct answer. Each = 2points, Total = 42 points**)

(i) One of the following binary relations on $A = \{1, 2\}$ is a lattice.

a) $\{(2, 2), (1, 1), (1, 2)\}$ b) $\{(2, 2), (1, 1), (1, 2), (2, 1)\}$. c) $\{(1, 2)\}$ d) $\{(1, 2), (2, 1), (1, 1)\}$

(ii) Let T be a partial order relation on $\mathbb{N}$ (recall $\mathcal{N} = \{1, 2, 3, 4, ...\}$) such that for every $a, b \in \mathcal{N}$, $(a, b) \in T$ iff $b \leq a$. Then $5 \bigvee 10 =$

a) 10 b) 5 c) Does not exist d) 1

(iii) Let T and $\mathbb{N}$ as in (ii). The maximum element of $\mathbb{N}$ under T is

a) Does not exist b) 1 c) Need more information d) None of the previous is correct

(iv) Let $A = \{0, 1, \{1\}, \{0\}\}$. Then $|P(A)| =$

a) 4 b)3 c) 8 d) 16

(v) Let A as in (iv). Then

a) $\{0\} \in A$ b) $\{1\} \in P(A)$ c) (a) and (b) are correct d) $\{0, 1\} \subset P(A)$ e) ALL previous statements are correct.

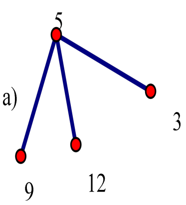
(vi) Let $G(V, E)$ be a Hamilton graph such that $|V| = 6$. Then

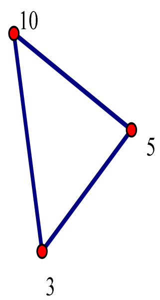
a) $deg(v) \geq 3$ for each $v \in V$. b) $|E| = 6$ c) $|E| \geq 6$ d) (a) and (b) are correct e) (a) and (c) are correct.

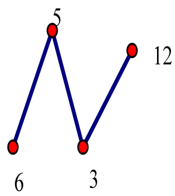
(vii) Let $F = \mathbb{Q} \cap [1, 6]$. Then

a) There is a bijection function from F onto $\mathcal{N}$. b) There is a bijection function from F onto $[1, 6]$ c) There is a bijection function from F onto $\mathbb{Q}$ d) (a) and (c) are correct. e) None of the previous is correct.

(viii) One of the following is a subgraph of $G(Z_{15})$

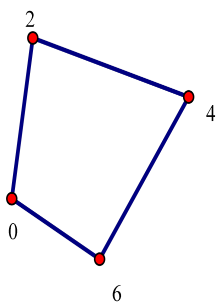
a) 

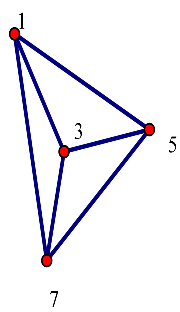
b) 

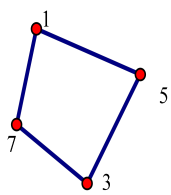
c) 

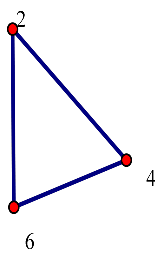
d) None is correct

(ix) One of the following is a component of $TG(Z_8)$.

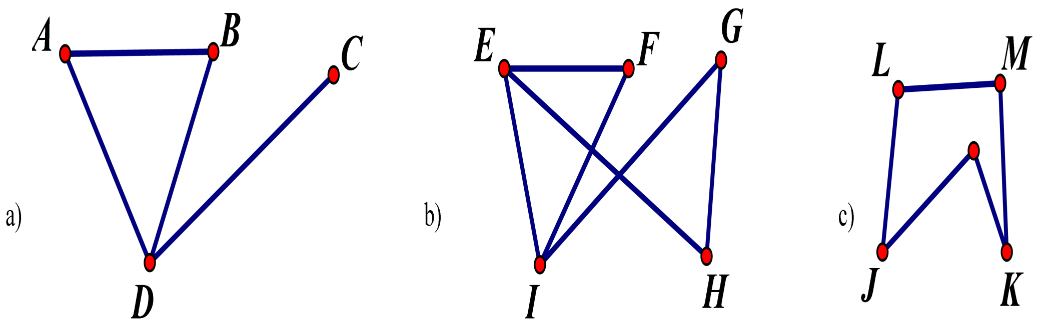
a) 

b) 

c) 

d) 

(x) One of the following is a Hamilton graph that is not an Euler graph



(xi) Let $x \in Z$. The set of solution of $8x \equiv 4(mod\ 10)$ over Z is

a) $\{3 + 10k \mid k \in Z\}$ b) $\{3, 8\}$ c)$\{3 + 5k \mid k \in Z\}$ d) None of the previous

(xii) Let $x \in Z_9$. The set of solution of $6x = 3$ over Z_9

a) $\{2\}$ b) $\{2, 5, 8\}$ c) $\{1/2\}$ d) None of the previous.

(xiii) Let $x \in Z$. The set of solution to $x \equiv 3(mod\ 6)$ and $x \equiv 3(mod\ 4)$.

a) $\{3 + 12k \mid k \in Z\}$ b) $\{15 + 24k \mid k \in Z\}$ c) $\{3, 15\}$ d) $\{3 + 24k \mid k \in Z\}$ e) None of the previous.

(xiv) Let $T(V, E)$ be a full 8-ary tree. Then one of the following is a possibility for $|V|$

a) 16 b) 21 c)8 d) 33 e) 15

(xv) $(11)_4 = (x)_5$. Then $x =$

a) 0 b) 10 c) 11 d) 01

(xvi) The graph $G(Z_{10})$ is

a) K_5 b) full 4-ary tree c) $K_{2,3}$ d) Euler but not Hamilton

(xvii) Let $n = 8 \times 44$ and $F = \{a \mid 1 \leq a \leq n$ and $gcd(a, n) = 4\}$. Then $|F| =$

a) 40 b) 77 c) 160 d) none of the previous

(xviii) Let $x \in Z$. The set of solution to $x \equiv 6(mod\ 7)$ and $x \equiv 5(mod\ 6)$ is

a) $\{13 + 42k \mid k \in Z\}$ b) $\{41 + 42k \mid k \in Z\}$ c) $\{1 + 13k \mid k \in Z\}$ d) None of the previous

(xix) If we divide 5^{146} by 13. Then the remainder is

a) 12 b) 1 c) 0 d) 5 e) None of the previous

(xx) $(101)_4 = (x)_{64}$. Then $x =$

a)$\underline{11}$ b) 11 c)$\underline{17}$ d) $\underline{20}$ e) None of the previous.

(xxi) One of the following is both Euler and Hamilton graph

a) K_8 b) $K_{4,2}$ c) $K_{5,5}$ d) K_9.

QUESTION 3. (**8 points**) Use Math Induction to prove that 3 is a factor of $n^3 + 8n + 6$ for each $n \geq 1$ (i.e., prove that $3 \mid (n^3 + 8n + 6)$)

QUESTION 4. (**6 points**) Show that $|R| = |(2, 4)|$ by constructing a bijection function from R onto $(2, 4)$. (Only write down a function that will do the job and graph it).

QUESTION 5. (**4 points**) Let $F = (1, 9] - \{5\}$ (i.e., F is the set of all real numbers between 1 and 9 except the numbers 1 and 5). Show that $|(0, 4)| = |F|$ by constructing a bijection function from (0, 4) onto F. (Only write down a function that will do the job and graph it).

QUESTION 6. (**10 points**) Given $a_0 = 3$, $a_1 = -2$, and $a_n = a_{n-1} + 12a_{n-2}$. Find a mathematical equation for a_n for each $n \geq 2$. Then find the 10^{th} term in the sequence.

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