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Exam I MTH 213 , Fall 2011

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QUESTION 1. (21 points, each = 1.5 points) Just write down T or F

(i) $\sqrt{15}$ is irrational number

(ii) π is irrational number

(iii) 3.14 is a rational number

(iv) $22/7$ is a rational number

(v) $-9(mod-13)$ is 4

(vi) $-9(mod 13)$ is 4

(vii) since $\frac{a}{b} = \frac{-a}{-b}$ for every nonzero positive integers a, b, we have $(-a)(mod - b) = a(mod\, b)$.

(viii) If x is a rational number, then $x + 1.7$ is a rational number

(ix) If $A = \{1, 3, 5\}$. Then $T = \{(1, 1),(5, 5),(3, 3),(1, 3),(5, 1)\}$ is a total order on A

(x) If $|A| = 21$ and T is a partial order on A, then $|T| \geq 21$

(xi) Let $A = \{4, 5, 7\}$ and $T = \{(4, 4)\}$. Then T is symmetric and transitive.

(xii) Let $A = \{0, 2, 7\}$ and $T = \{(0, 0),(2, 2),(7, 7),(2, 7),(7, 2)\}$. Then T is an equivalence relation such that A (under T) has exactly 2 distinct equivalent classes.

(xiii) If a relation T on a set A is not anti-symmetric, then T is symmetric.

(xiv) Assume A is a set with 3 elements, and $B = \{d \subset P(A) \mid |d| = 3\}$. Then $|B| = 56$.

QUESTION 2. (9 points, each = 1point) Let $A = \{0, \{6\}, \{0\}, \{0, 6\}, 7\}$. Then write down T or F

(i) $\{7\} \subset P(A)$

(ii) $\{0\} \in A$

(iii) $\{0\} \in P(A)$

(iv) A is a countable set.

(v) $\{\{0, 6\}, 7\} \in P(A)$

(vi) $|P(A)| = 32$

(vii) It is possible to have a binary relation T on A such that $|T| = 32$.

(viii) Let $K = \{x \in P(A) \mid |x| = 2\}$ and $F = \{y \in P(A) \mid |y| = 3\}$. Then there is a bijection function from K into F.

(ix) Let $K = \{x \in P(A) \mid |x| = 2\}$ and $F = \{y \in P(A) \mid |y| = 1\}$. Then there is a bijection function from K into F.

QUESTION 3. (8 points) Prove that $(A \cup B)^c = A^c \cap B^c$.

QUESTION 4. (7 points) Show that $|(2,8]| = |[4,1)|$ by constructing a bijection function from $(2,8]$ into $[4,1)$

QUESTION 5. (7 points) Let $D = Q^+ \cap (0,1)$. Find $|D|$ (explain your answer in at most 1.5 lines)

QUESTION 6. (10 points) Let $A = \{1,2,3,5,7,10\}$. Define a relation T on A such that whenever $a, b \in A$, $aTb \Leftrightarrow b = ak$ for some $k \in A$.

a) Find T

b) If T is a partial order on A, then find

(i) $5 \bigwedge 2$

(ii) $10 \bigvee 2$

(iii) If possible find the minimum element and the maximum element of A under T

QUESTION 7. (10 points) a) Let $A = \{6, 9, 1\}$. Construct a partial order relation on A, say T, such $6 \bigwedge 9 = 1$.

b) Is the relation T you constructed in (a) a lattice? Briefly explain (not more than a line)

c) If possible find the minimum element and the maximum element of A under T as in (a).

d) Can we construct a relation H on A (the same A as in (a)) such that H is a total order and $6 \bigvee 1 = 9$? If yes, then construct H. If no, then briefly explain

QUESTION 8. (10 points) Let $N = \{1, 2, 3, ..., \}$ be the set of all natural numbers, and let T be a binary relation on $P(N)$ such that whenever $x, y \in P(N) xTy \Leftrightarrow y \subseteq x$. We know that T is a partial order on $P(N)$.

(i) Find $\{34, 0, 1\} \bigvee \{77\}$

(ii) Find $\{7, 5, 3\} \bigwedge \{5, 3\}$

(iii) Is T a total order? Explain briefly

(iv) If possible find the minimum element and the maximum element of $P(N)$ under T.

QUESTION 9. (9 points) We know that Q^+ is countable. Use the algorithm we discussed in the class in order to find the next 10 numbers after 14,

QUESTION 10. (9 points) Let $A = \{1, 2, 3, 4\}$.

a) Construct an equivalence relation on A such that $D = \{1, 4\}$ and $F = \{2, 3\}$ are equivalent classes of A.

b) Construct an equivalence relation on A such that A has exactly 4 distinct equivalent classes

Faculty information

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