your hand-writing is good!! 99 Excellent

12:30

TEST NUMBER TWO FOR MTH221 SPRING 2007

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Name MO'OUD NASR, Id. Num. 16101, Score $\frac{}{100}$

QUESTION 1. (**15 points**) *After a long day at AUS, in your way back home driving your air-conditioned car, you heard the AUS Math. radio station π.13. Explain CLEARLY in at most two lines the meaning of the following statements that you heard on the radio:*

(1) $1+x^2-x^3$ *is not in the span of* $2+x^2$ *and* x^3

The polynomial $(1+x^2-x^3)$ cannot be written as a linear combination of $(2+x^2)$ and (x^3) : $1+x^2-x^3 \neq \alpha_1(2+x^2)+\alpha_2(x^3) \rightarrow$ There is no solution for α_1 & α_2

(2) *-3 belongs to the span of* $\pi+x^7$ *and* $e^\pi x^7$

-3 can be written as a linear combination of polynomials $(\pi+x^7)$ and $(e^\pi x^7)$: $-3 = \alpha_1(\pi+x^7)+\alpha_2(e^\pi x^7)$
There exists a solution for α_1 & α_2.

(3) $\{v_1, v_2, v_3\}$ *form a basis for* R^3.

v_1, v_2 and v_3 are 3 independent elements "living" in $\mathbb{R}^3$
The span of these 3 elements is equal to $\mathbb{R}^3$
span $\{v_1, v_2, v_3\} = \mathbb{R}^3$

(4) S *is a subspace of* R^4

S is a vectorspace "living" in $\mathbb{R}^4$ (it is a subset of $\mathbb{R}^4$ and it is a vector space. It satisfies the two conditions: 1) For every $v_1 \in S$ and $v_2 \in S \Rightarrow v_1+v_2 \in S$
2) For every $\alpha \in \mathbb{R}$ and $v \in S \Rightarrow \alpha v \in S$

(5) D *is not a subspace of* R^7

D might be a subset of $\mathbb{R}^7$ but it is <u>not</u> a vectorspace that resides in $\mathbb{R}^7$ also the additive identity of $\mathbb{R}^7$ does not exist in D.

(6) v_1, v_2, v_3 *are dependent elements in* R^3.

at least one of the 3 elements relies on the other and can be written as a linear combination of the other two.

5) It also does <u>NOT</u> satisfy at least one of the following conditions,
1) For every $v_1 \in D$ and $v_2 \in D \Rightarrow v_1+v_2 \in D$
2) For every $\alpha \in \mathbb{R}$ and $v_1 \in D \Rightarrow \alpha v_1 \in D$.

(7) *any 4 independent elements in R^4 form a basis for R^4.*

You need exactly 4 independent elements that live in $\mathbb{R}^4$ to form a basis of $\mathbb{R}^4$. There are infinitly many Basis's for $\mathbb{R}^4$. The span of those 4 independent elements is equal to $\mathbb{R}^4$.

(8) *$\dim(row(A))$ always equal $\dim(Col(A))$*

The number of independent Rows of matrix A is always equal to the number of independent columns of A which is equal to the rank of A.

The minimum no. of elements needed to span the rows of A are always equal to the min. no of elements needed to span columns of A.

(9) *Give me a hint on how to form a basis for $R^{3\times2}$.* $\mathbb{R}^{3\times 2} \cong \mathbb{R}^6$ and $\dim(\mathbb{R}^6) = \dim(\mathbb{R}^{3\times2}) = 6$

Set of all 3×2 matrices is isomorphic as vector space to $\mathbb{R}^6$.

Any 6 independent elements in $\mathbb{R}^6$ corresponding to the 6 independent elements in $\mathbb{R}^{3\times2}$ form a basis for $\mathbb{R}^{3\times 2}$.

(10) *3 is an eigenvalue of a 2×2 matrix A corresponds to the vector* $v = \begin{bmatrix}2\\3\end{bmatrix}$

$A\begin{bmatrix}2\\3\end{bmatrix} = 3\begin{bmatrix}2\\3\end{bmatrix}$

any linear combination of $\begin{bmatrix}2\\3\end{bmatrix}$ will satisfy the following equation:

$AV = 3V$

QUESTION 2. (**10 points**) *Let* $F = \{\begin{bmatrix}a\\a+c\\c\end{bmatrix} \mid a, c \in R\}$

(1) *Show that F is a subspace of R^3*

1) $v_1 = \begin{bmatrix}x\\x+y\\y\end{bmatrix} \in F$ $\quad v_2 = \begin{bmatrix}w\\w+z\\z\end{bmatrix} \in F$ $\quad v_1+v_2 = \begin{bmatrix}x+w\\x+y+w+z\\y+z\end{bmatrix} = \begin{bmatrix}x+w\\(x+w)+(y+z)\\y+z\end{bmatrix} \in F$

2) For $\alpha \in \mathbb{R}$ and $v_1 = \begin{bmatrix}x\\x+y\\y\end{bmatrix} \in F \Rightarrow \alpha v_1 = \begin{bmatrix}\alpha x\\\alpha(x+y)\\\alpha y\end{bmatrix} = \begin{bmatrix}\alpha x\\\alpha x+\alpha y\\\alpha y\end{bmatrix} \in F$

also $\begin{bmatrix}0\\0\\0\end{bmatrix} \in \mathbb{R}^3$ and $\begin{bmatrix}0\\0\\0\end{bmatrix} \in F \Rightarrow$ F is a subspace of $\mathbb{R}^3$

(2) *find a basis for F.*

$V = \begin{bmatrix}a\\a+c\\c\end{bmatrix}$ (2unknowns) $a, c \in \mathbb{R}$ (2 free variables) $\Rightarrow \dim(F) = 2$

(3) *Use part (2) to rewrite F.*

1st element: $a=1$, $c=0$ $\begin{bmatrix}1\\1\\0\end{bmatrix}$

2nd element: $a=0$, $c=1$ $\begin{bmatrix}0\\1\\1\end{bmatrix}$

Basis for $F = \left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

$F = \text{span}\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

$$\begin{bmatrix} 1-a_{21} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11} = 1 - a_{21}$

QUESTION 3. (**6 points**) *Let* $D = \{A \in R^{2\times 2} \mid a_{11} + a_{21} = 1\}$. *Is* D *a subspace of* $R^{2\times 2}$*? If no, then tell me why not!! if yes, then find dim(D).*

additive identity of $\mathbb{R}^{2\times 2}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{2\times 2}$ but $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin D$ $\therefore$ D is NOT a subspace of $\mathbb{R}^{2\times 2}$

QUESTION 4. (**14 points**) *Let* $M = \{f(x) \in P_4 \mid \int_0^1 f'(x)\,dx = 0\}$.

(1) *Show that* M *is a subspace of* P_4.

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ $\quad f'(x) = a_1 + 2a_2 x + 3a_3 x^2$

$$\int_0^1 (a_1 + 2a_2 x + 3a_3 x^2)\,dx = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \Big]_0^1 = 0$$

$\Rightarrow \cancel{a_0} + a_1 + a_2 + a_3 - \cancel{a_0} = 0 \Rightarrow \boxed{a_1 + a_2 + a_3 = 0} \Rightarrow \boxed{a_1 = -a_2 - a_3}$

$k(x) = a_0 + (-a_2 - a_3)x + a_2 x^2 + a_3 x^3$ condition

1) $Q(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 = a_0 + (-a_2 - a_3)x + a_2 x^2 + a_3 x^3 \in M$

$L(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 = b_0 + (-b_2 - b_3)x + b_2 x^2 + b_3 x^3 \in M$

$Q(x) + L(x) = (a_0 + b_0) + (-a_2 - a_3 - b_2 - b_3)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3$

$= \overbrace{(a_0 + b_0)}^{c_0} + (-\overbrace{(a_2 + b_2)}^{c_2} - \overbrace{(a_3 + b_3)}^{c_3})x + \overbrace{(a_2 + b_2)}^{c_2} x^2 + \overbrace{(a_3 + b_3)}^{c_3} x^3$

(2) *Give me some elements in M so that their span equals M.*

$\Rightarrow Q(x) + L(x) = c_0 + (-c_2 - c_3)x + c_2 x^2 + c_3 x^3 \in M$

2) For $\alpha \in \mathbb{R}$ and $Q(x) = a_0 + (-a_2 - a_3)x + a_2 x^2 + a_3 x^3 \in M$

$\alpha Q(x) = \alpha a_0 + \alpha(-a_2 - a_3)x + \alpha a_2 x^2 + \alpha a_3 x^3$

$= \overbrace{\alpha a_0}^{c_0} + (-\overbrace{\alpha a_2}^{c_2} - \overbrace{\alpha a_3}^{c_3})x + \overbrace{\alpha a_2}^{c_2} x^2 + \overbrace{\alpha a_3}^{c_3} x^3 = c_0 + (-c_2 - c_3)x + c_2 x^2 + c_3 x^3 \in M$

$\therefore$ M is a subspace of P4 also, $(0,0,0,0) \in M$ and P4

$P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ $\quad P(x) = a_0 + (-a_2 - a_3)x + a_2 x^2 + a_3 x^3$

4 unknowns $\quad a_1 = -a_2 - a_3 \quad$ free variable: $a_2, a_3, a_0 \in \mathbb{R}$

$\dim(M) = 3$

1st polynomial: $a_2 = 1 \quad a_3 = a_0 = 0 \Rightarrow -x + x^2$

2nd polynomial: $a_3 = 1 \quad a_2 = a_0 = 0 \Rightarrow -x + x^3$

3rd polynomial: $a_0 = 1 \quad a_2 = a_3 = 0 \Rightarrow 1$

Basis $= \{1, -x + x^2, -x + x^3\}$

$M = \text{Span}\{1, -x + x^2, -x + x^3\}$

$$\begin{bmatrix} 1-a_{21} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11} = 1 - a_{21}$

QUESTION 3. *(6 points) Let $D = \{A \in R^{2\times2} \mid a_{11} + a_{21} = 1\}$. Is D a subspace of $R^{2\times2}$? If no, then tell me why not!! if yes, then find dim(D).*

additive identity of $\mathbb{R}^{2\times2}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{2\times2}$ but $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \notin D$ $\therefore$ D is NOT a subspace of $\mathbb{R}^{2\times2}$

QUESTION 4. *(14 points) Let $M = \{f(x) \in P_4 \mid \int_0^1 f'(x)\, dx = 0\}$.*

(1) *Show that M is a subspace of P_4.*

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ $\qquad f'(x) = a_1 + 2a_2x + 3a_3x^2$

$\int_0^1 (a_1 + 2a_2x + 3a_3x^2)\,dx = a_0 + a_1x + a_2x^2 + a_3x^3 \Big]_0^1 = 0$

$\Rightarrow \cancel{a_0} + a_1 + a_2 + a_3 - \cancel{a_0} = 0 \Rightarrow \boxed{a_1 + a_2 + a_3 = 0} \Rightarrow \boxed{a_1 = -a_2 - a_3}$ condition

$k(x) = a_0 + (-a_2 - a_3)x + a_2x^2 + a_3x^3$

1) $Q(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = a_0 + (-a_2 - a_3)x + a_2x^2 + a_3x^3 \in M$

$L(x) = b_0 + b_1x + b_2x^2 + b_3x^3 = b_0 + (-b_2 - b_3)x + b_2x^2 + b_3x^3 \in M$

$Q(x) + L(x) = (a_0 + b_0) + (-a_2 - a_3 - b_2 - b_3)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3$

$= \overbrace{(a_0 + b_0)}^{c_0} + (-\overbrace{(a_2 + b_2)}^{c_2} - \overbrace{(a_3 + b_3)}^{c_3})x + \overbrace{(a_2 + b_2)}^{c_2}x^2 + \overbrace{(a_3 + b_3)}^{c_3}x^3$

(2) *Give me some elements in M so that their span equals M.*

$\Rightarrow Q(x) + L(x) = c_0 + (-c_2 - c_3)x + c_2x^2 + c_3x^3 \in M$

2) For $\alpha \in \mathbb{R}$ and $Q(x) = a_0 + (-a_2 - a_3)x + a_2x^2 + a_3x^3 \in M$

$\alpha Q(x) = \alpha a_0 + \alpha(-a_2 - a_3)x + \alpha a_2x^2 + \alpha a_3x^3$

$= \overbrace{\alpha a_0}^{c_0} + (-\overbrace{\alpha a_2}^{c_2} - \overbrace{\alpha a_3}^{c_3})x + \overbrace{\alpha a_2}^{c_2}x^2 + \overbrace{\alpha a_3}^{c_3}x^3 = c_0 + (-c_2 - c_3)x + c_2x^2 + c_3x^3 \in M$

$\therefore$ M is a subspace of P4 also, $(0,0,0,0) \in M$ and P4

$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ 4 unknowns.

$P(x) = a_0 + (-a_2 - a_3)x + a_2x^2 + a_3x^3$

$a_1 = -a_2 - a_3$ $\qquad$ free variable: $a_2, a_3, a_0 \in \mathbb{R}$

dim(M) = 3

1st polynomial: $a_2 = 1$ $\quad a_3 = a_0 = 0 \Rightarrow -x + x^2$

2nd polynomial: $a_3 = 1$ $\quad a_2 = a_0 = 0 \Rightarrow -x + x^3$

3rd polynomial: $a_0 = 1$ $\quad a_2 = a_3 = 0 \Rightarrow 1$

Basis = $\{1, -x + x^2, -x + x^3\}$

$M = \text{Span}\{1, -x + x^2, -x + x^3\}$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad Q = \begin{bmatrix} 0 & -2 & -3 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

eigen values

QUESTION 6. **(15 points)** *Let* $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$ *If possible find a nonsingular matrix* Q *and a diagonal matrix* D *such that* $Q^{-1}AQ = D$.

$$\det(A - \alpha I_n) = \det\left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}\right) = \det\left(\begin{bmatrix} -1-\alpha & 0 & 0 \\ 0 & -1-\alpha & 0 \\ 1 & 2 & 2-\alpha \end{bmatrix}\right)$$

$$= (-1-\alpha)(-1)^2 \det\begin{bmatrix} -1-\alpha & 0 \\ 2 & 2-\alpha \end{bmatrix} + 0 + 0 = (-1-\alpha)\left[(-1-\alpha)(2-\alpha) - 0\right] = (-1-\alpha)^2(2-\alpha) = 0$$

$2 - \alpha = 0 \Rightarrow \boxed{\alpha = 2}$ $\quad -1-\alpha = 0 \Rightarrow \boxed{\alpha = -1}$

$$E_2 = N(A - 2I_n) = N\left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right) = N\left(\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 1 & 2 & 0 \end{bmatrix}\right)$$

$$\left[\begin{array}{ccc|c} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right] \xrightarrow{-\frac{1}{3}R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right] \xrightarrow{-R_1+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right] \xrightarrow{-2R_2+R_3 \to R_3} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Solution: $x_2 = 0$, $x_1 = 0$, $x_3 \in \mathbb{R}$, $0 = 0$

QUESTION 7. **(5 points)** *Find a basis for* P_4 *that contains the following two independent polynomials:* $-2 + x, 1 + x^2$

↓ cont'd.

back Q 7)

6) $N(A) = \left\{ \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix} \middle| x_3 \in \mathbb{R} \right\} \Rightarrow$ basis for $E_2 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$E_{-1} = N(A + I_n) = N\left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = N\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 3 \end{bmatrix}\right)$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right]$$

$x_1 + 2x_2 + 3x_3 = 0 \Rightarrow x_1 = -2x_2 - 3x_3$ $\quad x_2$ and $x_3 \in \mathbb{R}$

$$N(A) = \left\{ \begin{bmatrix} -2x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} \middle| x_2, x_3 \in \mathbb{R} \right\}$$

Basis for $E_{-1} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$ $\qquad E_{-1} = \text{Span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

$x_2 = 1$, $x_3 = 0$ $\qquad x_3 = 1$, $x_2 = 0$

↑ above

Q7) $-2+x \longleftrightarrow \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$1+x^2 \longleftarrow \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \xrightarrow{\frac{1}{2}R_1+R_2 \to R_2} \sim \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\longleftrightarrow \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Basis for $P_4 = \{ -2+x, 1+x^2, 3x^2+4x^3, 5x^3 \}$

Check

$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix} \xleftrightarrow{\frac{1}{2}R_1+R_2 \to R_2} \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$\therefore$ chosen elements are independent.

Basis for $N(A) = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

QUESTION 8. *(15 points) Let* $A = \begin{bmatrix} 1 & 2 & 4 & 0 & 1 & -1 \\ -1 & -2 & -4 & 1 & 2 & 2 \\ -2 & -4 & -8 & 0 & -2 & 2 \end{bmatrix}$

(1) *Find a basis for Row(A).*

$$\begin{bmatrix} 1 & 2 & 4 & 0 & 1 & -1 \\ -1 & -2 & -4 & 1 & 2 & 2 \\ -2 & -4 & -8 & 0 & -2 & 2 \end{bmatrix} \xrightarrow[2R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \begin{bmatrix} 1 & 2 & 4 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis for $Row(A) = \left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -4 \\ 1 \\ 2 \\ 2 \end{bmatrix} \right\}$

Basis for $Col(A) = \left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

$$\left[\begin{array}{cccccc|c} 1 & 2 & 4 & 0 & 1 & -1 & 0 \\ -1 & -2 & -4 & 1 & 2 & 2 & 0 \\ -2 & -4 & -8 & 0 & -2 & 2 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left[\begin{array}{cccccc|c} 1 & 2 & 4 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$ reduced echelon

(2) *Find a basis for Col(A).*

Solution:

$0 = 0$

$x_4 + 3x_5 + x_6 = 0 \Rightarrow x_4 = -3x_5 - x_6$

(3) *Find a basis for N(A) (If you want finish it at the back of this page)*

$x_1 + 2x_2 + 4x_3 + x_5 - x_6 = 0 \Rightarrow x_1 = -2x_2 - 4x_3 - x_5 + x_6 \quad x_2, x_3, x_5, x_6 \in \mathbb{R}$

THERE WILL BE CLASS ON MONDAY. WE START CHAPTER 4.

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$N(A) = \left\{ \begin{bmatrix} -2x_2 - 4x_3 - x_5 + x_6 \\ x_2 \\ x_3 \\ -3x_5 - x_6 \\ x_5 \\ x_6 \end{bmatrix} \middle| \ x_2, x_3, x_5, x_6 \in \mathbb{R} \right\}$

$x_2 = 1$, $x_3 = x_5 = x_6 = 0$: $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $x_3 = 1$, $x_2 = x_5 = x_6 = 0$: $\begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $x_5 = 1$, $x_3 = x_2 = x_6 = 0$: $\begin{bmatrix} -1 \\ 0 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$

$x_6 = 1$, $x_2 = x_3 = x_5 = 0$ $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

Above