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Exam I: MTH 221, Fall 2015

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QUESTION 1. (i) Given A is a 10×10 matrix such that $det(A) = 0$. Let B be the second column of A and consider the system of linear equations $AX = B$. Then

a. The system has no solutions (inconsistent).
b. $\{(0, 1, 0, 0)\}$ is the solution set to the system.
~~c.~~ The system has infinitely many solutions.
d. None of the above is correct.

(ii) Let $A = \begin{bmatrix} 0 & a & b \\ 0 & -2 & c \\ 0 & 0 & 1 \end{bmatrix}$. Then $det(A + 3I_3) =$ is

(a) 9 (b) 27 ~~(c)~~ 12 (d) 3

(iii) Let $A = \begin{bmatrix} 2 & 4 & -1 & 2 \\ -3 & 7 & 11 & 23 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 22 \\ -4 & 3 & 15.2 \\ 9 & 0 & 77.5 \\ -7 & 0 & 88 \end{bmatrix}$. Let $D = AB$. Then the second column of D is

~~(a)~~ $\begin{bmatrix} 12 \\ 21 \end{bmatrix}$ (b) $\begin{bmatrix} 21 \\ 9 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 21 \end{bmatrix}$ (d) Something else

(iv) Given A is a 4×4 matrix and $A \xrightarrow{2R_1 + R_3 \to R_3} A_1 \xrightarrow{3R_4} \begin{bmatrix} 1 & 3 & 4 & -1 \\ 0 & 3 & 1 & 8 \\ 0 & -3 & 2 & 1 \\ -1 & -3 & -4 & 4 \end{bmatrix}$. Then $det(A^T)$

~~(a)~~ 9 (b) 27 (c) $\frac{1}{9}$ (d) 51

(v) Given A is a 2×3 matrix and $A \xrightarrow{-2R_1 + R_2 \to R_2} B \xrightarrow{4R_2} D = \begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \end{bmatrix}$. Let F, W be two elementary matrices such that $FWA = D$. Then

(a) $W = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, ~~(b)~~ $F = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ (c) $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) Something else

(vi) Given A is a 3×3 matrix and $A \xrightarrow{-2R_1 + R_2 \to R_2} B \xrightarrow{4R_2} D = \begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \\ 1 & 2 & 62 \end{bmatrix}$. Then

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A =$$

(a) $\begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \\ 1 & 2 & 62 \end{bmatrix}$ ~~(b)~~ $\begin{bmatrix} 1 & 3 & 66 \\ 8 & 12 & 21 \\ 0 & -1 & -4 \end{bmatrix}$. (c) D^T (d) Something else

(vii) Given A is nonsingular matrix such that $A^{-1} = \begin{bmatrix} a & -3 & 2 \\ b & 1 & 1 \\ c & 0 & -3 \end{bmatrix}$, for some fixed numbers a, b, c. Consider the system of linear equations $AX = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. Then

(a) It is possible that the system has no solution. ~~(b)~~ $\{(1, 3, -6)\}$ is the solution set of the system
(c) $\{(a-6, b+2, c)\}$ is the solution set of the system (d) Not enough information in order to determine the solution set, but I am sure that it must have a unique solution.

(viii) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$. Then A^{-1} =

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$ ~~(b)~~ $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ (c) There is no inverse of A (d) Something else

(ix) Given $A = \begin{bmatrix} a & 2 & 0 \\ b & 1 & 4 \\ c & 0 & d \end{bmatrix}$ such that $det(A) = -4$ (i.e, A is invertible). Then the (1, 3)-entry of A^{-1} is

(a) $\frac{c}{-4}$ (b) $\frac{c}{4}$ ~~(c)~~ -2 (d) something else

(x) Let $A = \begin{bmatrix} 2 & 4 & a \\ 0 & 2 & b \\ 0 & -2 & d \end{bmatrix}$ such that $det(A) = 2$. Consider the system $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$. Then the value of x_3

(a) cannot be determined, I need more info. (b)5 ~~(c)~~ 10 (d) 6 (e) Something else

(xi) Let $A = \begin{bmatrix} 2 & a & b \\ -2 & 4 & 7 \\ 4 & 2a & 10 \end{bmatrix}$. The values of a, b where the system $AX = \begin{bmatrix} \sqrt{7} \\ 2015 \\ 36.23 \end{bmatrix}$ has unique solution are :

~~(a)~~ $a \neq -4$ and $b \neq 5$ (b) $a \neq 0$ and b can be any real number (c) $a = 0$ and $b \neq 0$ or $b = 0$ and $a \neq 0$. (d) Something else

(xii) Given A is a 2×2 matrix such that $A \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} - 2A = I_2$. Then $A =$

~~(a)~~ $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (c) $\frac{1}{3} \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ (d) Something else

(xiii) Given the augmented matrix of a system of linear equations $\begin{bmatrix} 2 & 4 & 2 & 0 & 4 \\ -1 & -2 & -1 & 1 & -1 \\ 3 & 6 & 4 & 0 & 3 \\ -1 & -2 & -1 & 0 & -2 \end{bmatrix}$. The solution set of the system is

(a) $\{(5 - 2x_2,\ x_2,\ -3,\ 1) \mid x_2 \in R\}$ $\{(-1 + x_4,\ 1,\ -3,\ x_4) \mid x_4 \in R\}$ $\{(5,\ 0,\ -3,\ 1)\}$ (d) Something else

(xiv) Given the augmented matrix of a system of linear equations $\begin{bmatrix} 1 & -1 & 3 & 4 \\ -1 & 1 & a & 6 \\ -2 & 2 & b & -8 \end{bmatrix}$. The values of a, b that make the system consistent (i.e, has a solution)

~~(a)~~ $a \neq -3$ and $b = -6$ (b) a can be any real number, $b \neq -6$ (c) $a = -3$ and $b = -6$ (d) Something else.

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