Name Israa ID g00097435

Final Exam

Ayman Badawi

$\frac{58}{58}$

QUESTION 1. (i) **(6 points)** Let $A = \{3, 4, 12, 13, 15\}$, define "=" on A such that for every $a, b \in A$, a" = "b iff a $mod(9) = b$ $(mod\ 9)$. Then " = " is an equivalence relation on A. Find all equivalence classes of =. Assume that " = " is a subset of $A \times A$. WRITE DOWN ALL elements of " = ".

$[3] = \{3, 12\}$

$[4] = \{4, 13\}$

$[15] = \{15\}$

6

"=" $= \{(3,3), (3,12), (12,3), (12,12), (4,4), (4,13), (13,4), (13,13), (15,15)\}$

$8 \leqslant 16$

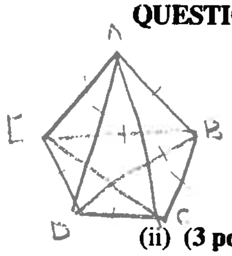
(ii) **(3 points)** Define "<=" on $A = \{1, 3, 8, 16\}$ such that $a \leq b$ iff $b = ac$ for some $c \in \{1, 2, 0.5, 10\}$. Then " <= " is not a partial order relation on A. Which axiom fails? expiain.

$8 \leqslant 16$ since $16 = 8 \times 2$, $2 \in C$

however, $16 \leqslant 8$ because $8 = 16 \times 0.5$ 3

$\therefore$ anti-symmetric axiom fails, and therefore this is not a partial order

QUESTION 2. (i) **(3 points)** Is K_5 a Hamiltonian? If yes construct a Hamiltonian cycle.



hamiltonian cycle: A – B – C – D – E – A

yes.

3

(ii) **(3 points)** Is K_5 an Eulerian circuit ? If yes, construct such circuit.

yes.

A – B – C – D – E – A – D – B – E – C – A 3

(iii) **(3 points)** Convince me that it is imopposible to construct a graph of order 8 so that the vertices have degrees 3, 3, 3, 2, 2, 2, 1, 1. [Hint: Use a beautiful result].

$|V| = 8$

fact: sum of all degrees of all vertices should be an even integer to satisfy $\frac{\Sigma \text{degrees}}{2} = |E|$

however, $3+3+3+2+2+2+1+1 = 17$. therefore, it is impossible to have a graph of order 8 with the given degrees.

3

(iv) (4 points) Let $a_n = 7a_{n-1} - 12a_{n-2} + 24$. Find a general formula for a_n. Do not find C_1, C_2

1) find c (the particular element)

$a_{p(n)} - 7a_{p(n-1)} + 12a_{p(n-2)} = 24$

$c - 7c + 12c = 24$

$\therefore c = 4$

2) find homogeneous solution

$$\frac{\alpha^n = 7\alpha^{n-1} - 12\alpha^{n-2}}{\alpha^{n-2}}$$

$\alpha^2 = 7\alpha - 12$

$\therefore \alpha = 4, 3$

3) combine

$a_n = c_1(4)^n + c_2(3)^n + 4$

QUESTION 3. **(i) (3 points)** The digits 1, 2, ..., 9 will be used to construct cars plates. How many car-plates can be constructed if adjacent digits must be different, the first digit must be odd and the last digit must be even ? Note that each plate consists of 6 digits.

9 numbers 2, 4, 6, 8 1, 3, 5, 7, 9

5 8 8 8 7 4 = 71680

(ii) (3 points) Let m be the number of candy-bags that will be distributed over 53 schools. What is minimum value of m so that a school will have at least 31 candy-bags.

$\left\lceil \frac{m}{53} \right\rceil = 31$

$m = 30 \times 53 + 1$

$= 1591$

(iii) (3 points) 302 positive integers, where each is of the form $3k$ for some integer k, are available. Then there are at least m integers out of the given 302 integers say $a_1, \ldots, a_m$ such that $a_1 \ (mod\ 12) = a_2 \ (mod 12) \cdots = a_m \ (mod\ 12)$. What is the best value of m?

0, 3, 6, 9, ...

3, 6, 9, 0

|domain| = 302 |codomain| = 4

$\{\vdots, 302\} \to \{0, 3, 6, 9\}$

$\left\lceil \frac{302}{4} \right\rceil = 76$

QUESTION 4. (6 points) *For* $k = 4$ *to* $(n^3 + 3)$ *do*

$x = y^3 + 7 * y + i - 4$

for $i = 1$ *to* k *do*

$z = x^2 + i^4 + i * x$

Next i 1 2 5 6 7

next k

a) Find the exact number of arithmetic operation executed by the code?

outer loop:

iterations: $(n^3+3) - 4 + 1 = n^3$

operations: $4n^3$

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inner loop:

iterations: $k - 1 + 1 = k$

operations: $7k$

→ first iteration: $7(4) = 28$

→ last iteration: $7(n^3+3)$

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total $= 4n^3 + \left[\dfrac{28 + 7(n^3+3)}{2}\right]\left[n^3\right]$

6/

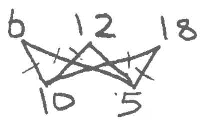
b) Find the complexity of the code, i.e. O(code).

$O(n^6)$

6 − 10

QUESTION 5. Let $V = \{6, 10, 12, \cancel{15}\ 5, 18\}$. Two vertices $v_1, v_2 \in V$ are connected by an edge if and only if $(v_1 v_2 \ mod\ 30) = 0$.

(i) **(3 points)** By drawing the graph, convince me that the graph is a $K_{m,n}$



$K_{3,2}$

M

(ii) **(3 points)** Is the graph an Eulerian Trail? If yes, construct such trail.

10 − 6 − 5 − 18 − 10 − 12 − 5 → yes, Eulerian trail

M

(iii) **(3 points)** If the graph is a Hamiltonian Path, construct such path.

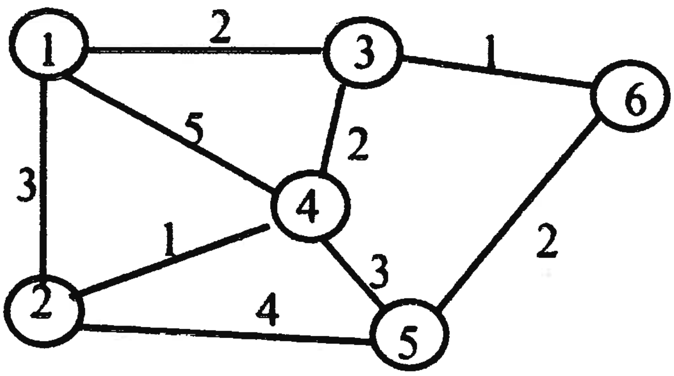
yes it is.

6 − 10 − 18 − 5 − 12

M

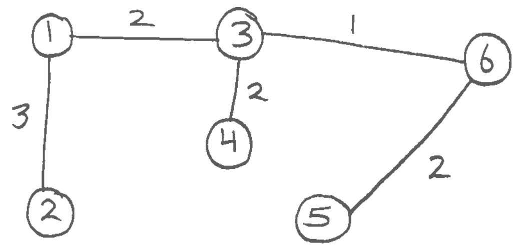
QUESTION 6. (8 points) Stare at the below picture.

Consider the network (nodes, links and their weights) in the figure below.



	1	2	3	4	5	6
1	$\boxed{0^1}$	3^1	2^1	5^1	∞	∞
3		3^1	$\boxed{2^1}$	4^3	∞	3^3
2		$\boxed{3^1}$		4^3	7^2	3^3
6				4^3	5^6	$\boxed{3^3}$
4				$\boxed{4^3}$	5^6	
5					$\boxed{5^6}$	

Use Dijkstra's Algorithm to construct the minimum weight spanning tree from vertex 1 to every other vertex.



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QUESTION 7. (6 points) Let x be the number of a particular candy in a bag. Given $1 \le x \le 88$. If $x \ (mod\ 8) = 5$ and $x \ (mod\ 11) = 6$. Find the value of x.

$m_1 = 8, \ m_2 = 11$

$r_1 = 5, \ r_2 = 6$

$m = 8 \times 11 = 88$

$n_1 = \frac{88}{8} = 11$

$n_2 = \frac{88}{11} = 8$

n_1^{-1} in $Z_{m_1} \longrightarrow (11)^{-1}$ in $Z_8 = 3$

n_2^{-1} in $Z_{m_2} \longrightarrow (8)^{-1}$ in $Z_{11} = 7$

$x = [(5)(11)(3) + (6)(8)(7)] (mod\ 88)$

$= 61$

6