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MTH 213, Discrete Math, Spring 2024, 1–3

Exam II

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QUESTION 1. (i) **(4 points)** Let $A = \{3, 6, 10, 40\}$ and $B = \{2, 4\}$ define "$\leq$" on A such that for every $a, b \in A$, $a \leq b$ iff $a = bc$ for some $c \in B$. Then "$\leq$" is not a partial order on A. Why?

6 "$\not\leq$" 6, since $6 = 6 \times 1$, and $1 \notin B$

$\therefore$ reflexive axiom does not hold, and therefore "$\leqslant$" is not a partial order. ✓

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(ii) **(4 points)** Define "=" on Z such that for every $a, b \in Z$, a" = "b iff a $mod(6) = b$ $mod(6)$. Then " = " is an equivalence relation on Z. Find all equivalence classes. If we view "=" as a subset of $A \times A$. What is the size of "=". ∞

$\bar{0} = \{\ldots -12, -6, 0, 6, 12, \ldots\}$

$\bar{1} = \{\ldots -11, -5, 1, 7, 13, \ldots\}$

$\bar{2} = \{\ldots -10, -4, 2, 8, 14 \ldots\}$

$\bar{3} = \{\ldots -9, -3, 3, 9, 15 \ldots\}$

$\bar{4} = \{\ldots -8, -2, 4, 10, 16 \ldots\}$

$\bar{5} = \{\ldots -7, -1, 5, 11, 17 \ldots\}$

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(iii) **(4 points)** Convince me that $\leq = \{(1,1), (2,2), (3,3), (1,3), (3,2)\}$ is not a partial order relation on $A = \{1, 2, 3\}$

since 1"$\leqslant$"3 and 3"$\leqslant$"2, we should have 1"$\leqslant$"2 to meet the transitive axiom, but that condition doesn't hold

$\therefore$ transitive axiom does not hold, and therefore "$\leqslant$" is not a partial order.

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(iv) **(4 points)** Given " = " = $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,3),(3,1),(2,6),(6,2)\}$ is an equivalence relation. Find all equivalence classes of "=".

$\bar{1} = \{1,3\}$

$\bar{2} = \{2,6\}$

$\bar{4} = \{4\}$

$\bar{5} = \{5\}$

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QUESTION 2. (i) **(4 points)** Let $a_n = 6a_{n-1} - 9a_{n-2}$ such that $a_1 = 6$ and $a_2 = 27$. Find a general formula for a_n.

$$\frac{\alpha^n = 6\alpha^{n-1} - 9\alpha^{n-2}}{\alpha^{n-2}}$$

$\alpha^2 = 6\alpha - 9$

$\therefore \alpha = 3$

$\rightarrow a_n = c_1(3)^n + c_2(3)^n n$

$6 = c_1(3) + c_2(3)$

$27 = c_1(3)^2 + c_2(3)^2(2)$

$\therefore c_1 = 1,\ c_2 = 1$

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general formula: $a_n = (3)^n + (3)^n n$

(ii) **(6 points)** Let $a_n = 5a_{n-1} - 6a_{n-2} + 10$, $a_1 = 4$ and $a_2 = 10$. Find a general formula for a_n.

find c:

$a_{p(n)} - 5a_{p(n-1)} + 6a_{p(n-2)} = 10$

$c - 5c + 6c = 10$

$\therefore c = 5$

find homogeneous solution:

$$\frac{\alpha^n = 5\alpha^{n-1} - 6\alpha^{n-2}}{\alpha^{n-2}}$$

$\rightarrow \alpha^2 = 5\alpha - 6$

$\therefore \alpha = 3, 2$

$a_n = c_1(3)^n + c_2(2)^n + 5$

$4 = c_1(3) + c_2(2) + 5$

$10 = c_1(3)^2 + c_2(2)^2 + 5$

$\therefore c_1 = \frac{7}{3},\ c_2 = -4$

$a_n = \frac{7}{3}(3)^n - 4(2)^n + 5$

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(iii) **(4 points)** The digits 1, 2, ..., 9 will be used to construct cars plates. How many ODD number plates can be constructed if repetition is not allowed?

8 7 6 5 4 5 $= 8 \times 7 \times 6 \times 5 \times 4 \times 5 = 33,600$

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(1) 2 (3) 4 (5) 6 (7) 8 (9)

QUESTION 3. (i) **(4 points)** Let m be the number of balls that will be distributed over 21 schools. What is minimum value of m so that a school will have at least 16 balls.

$\left\lceil \frac{m}{21} \right\rceil = 16$

$m = 21 \times 15 + 1$

$= 316$

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(ii) **(4 points)** 720 positive EVEN integers are available. Then there are at least m even integers out of the given 720 even integers say $a_1, ..., a_m$ such that $a_1 \ (mod\ 7) = a_2 \ (mod 7) \cdots = a_m \ (mod\ 7)$. What is the best value of m?

0, 2, 4, 6, 1, 3, 5,

$m = \left\lceil \frac{720}{7} \right\rceil = 103$

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QUESTION 4. *For* $k = 2$ *to* $(n^2 - 1)$ *do*
$x = y^3 + 7 * y + 4^3 - y$ $\longrightarrow 8$
for $i = 1$ *to* k *do*
$z = x^2 + i^3 - i * x + 2$ $\longrightarrow 7$
Next i
next k
a) Find the exact number of arithmetic operation executed by the code?

outer loop:

iterations: $(n^2-1) - 2 + 1 = n^2 - 2$

operations: $8(n^2 - 2)$

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inner loop:

iterations: $k - 1 + 1 = k$

operations: $7k$

$\rightarrow$ first iteration $= 7(2) = 14$

$\rightarrow$ last iteration $= 7(n^2 - 1)$

$\therefore$ total arithmetic operations: $8(n^2 - 2) + \left[\frac{14 + 7(n^2-1)}{2}\right]$ $[n^2 - 2]$

b) Find the complexity of the code, i.e. O(code).

$O(n^4)$