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Final Exam

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QUESTION 1. (i) Let $T : R^2 \to R^2$ be a L. T such that $T(1,0) = (4,6)$ and $T(0,1) = (6,12)$. The standard matrix presentation of T, M, is

(a) $M = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$ (b) $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $M = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$ (d) $M = \begin{bmatrix} 4 & 6 \\ 6 & 12 \end{bmatrix}$

(ii) Let $T : R^2 \to R$ be a L. T such that $T(1,1) = 3$ and $T(0,3) = 6$. Then $T(1,7) =$

(a) 14 (b) 10 (c) 15 (d) 13

(iii) Let $T : R^3 \to R^2$ such that $T(x_1, x_2, x_3) = (x_1 - 2x_3, -2x_1 + 4x_3)$. The set of all points (vectors) in R^3 where $T(x_1, x_2, x_3) = (5, -10)$ is

(a) Span$\{(5,0,0)\}$ (b) $\{(5 + 2x_3,\ x_2,\ x_3) \mid x_2, x_3 \in R\}$ (c) span$\{(5,0,0), (2,0,1), (0,1,0)\}$

(d) $\{(5 + 2x_3,\ 0,\ x_3) \mid x_3 \in R\}$ (e) span$\{(2,0,1), (0,1,0)\}$

(iv) Let T as in (iii). The $Ker(T) =$

(a) Span$\{(2,0,1), (0,1,0)\}$ (b) span$\{(0,1,0)\}$ (c) $\{(2x_3,\ 0,\ x_3) \mid x_3 \in R\}$ (d) span$\{(2,1,1)\}$

(v) One of the following is a linear transformation from R^3 to R^2

(a) $T(x_1, x_2, x_3) = (-x_1 + 3x_2,\ 7x_1 + 5x_3)$ (b) $T(x_1, x_2, x_3) = (x_2,\ 4)$

(c) $T(x_1, x_2, x_3) = (0, x_1x_3)$ (d) $T(x_1, x_2, x_3) = (2 + x_1,\ x_2)$

(vi) Let $D = span\{x^2 - x - 1, x^2,\ 3x^2 - x - 1\}$. A basis for D is

(a) $B = \{x^2 - x - 1,\ x^2\}$ (b) $B = \{x^2 - x - 1,\ x^2,\ 3x^2 - x - 1\}$

(c) $B = \{x^2 - x - 1, -x + 2\}$ (d) $B = \{x^2 - x - 1\}$

(viii) Let $D = \{(a-2b)x^2 + (2a-4b)x \mid a, b \in R\}$. Then $dim(D) =$

(a) 2 (b) 4 (c) 1 (d) 3

(ix) Let $A = \begin{bmatrix} 3 & 13 \\ 1 & 4 \end{bmatrix}$. Then A^{-1}

(a) $\begin{bmatrix} 3 & -13 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -13 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 13 \\ 1 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 13 \\ 1 & -3 \end{bmatrix}$

(x) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Then $|A + 4I_3| =$

(a) 65 (b) 5 (c) 125 (d) 9

(xi) Let $T : R^2 \to R^2$ be a linear transformation such that $T(a_1, a_2) = (2a_1 + a_2,\ 8a_1 + 4a_2)$. Then the eigenvalues of A are

(a) -2, -4 (b) 2, 3 (c) 0, 6 (d) 2, 4

(xii) Consider the following system $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_2 + a_3 \\ b_2 + b_3 \\ c_2 + c_3 \end{bmatrix}$, where $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 2024$. Then

(a) The system has no solutions (b) the system has infinitely many solutions and (0, 1, 1) is a solution.

(c) (0, 1, 1) is the only solution for the system.

QUESTION 2. Let $B = \{(1,1,1), (-1,0,0), (-1,-1,0)\}$ be a basis for R^3. Find the coordinate matrix presentation of R^3 with respect to B.

Find $[(2,1,-1)]_B$

Let

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

If A is diagnolizable, then find D, Q such that $A = QDQ^{-1}$.

QUESTION 3. Let $D = span\{1, 3x^2 + 1\}$. Define an inner product on D such that $< f_1, f_2 >= \int_0^1 f_1 f_2 \, dx$. Use Gram-Schmidt and find an orthogonal basis for D.

QUESTION 4. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 \\ -2 & -2 & -2 & -2 \end{bmatrix}$. Find a basis for Col(A).

Faculty information

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