# Exam III

## Ayman Badawi

**QUESTION 1.** (**Show the work, 12 points**)

Given $A$ is a $2 \times 2$ matrix where $2, -2$ are the only eigenvalues of $A$. Let $B = A^2 + 4A^{-1} + I_2$.

(i) Find the eigenvalues of $B$.

(ii) Find $|B|$.

(iii) Find $Trace(B)$.

(iv) Find $A^4$.

**QUESTION 2.** (**Show the work, 9 points**) Let

$$A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$$

If $A$ is diagnolizable, then find a diagonal matrix $D$ and invertible matrix $Q$ such that $Q^{-1}AQ = D$.

**QUESTION 3.** (**Show the work, 9 points**) Let $D = \{a_3x^3 + a_2x^2 + a_1x + a_0 \mid a_3 + a_2 + a_1 + a_0 = 0\}$ Show that $D$ is a subspace of $P_4$. Find a basis for $D$. Find dim(D).

**QUESTION 4.** (**Show the work, 9 points**)

Let $D = \{A \in R^{2 \times 2} \mid A^T = -A\}$. Show that $D$ is a subspace of $R^{2 \times 2}$. Find a basis for $D$ and dim(D).

**Faculty information**

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