

The right answer is a for  
all 14 questions

### Exam I

The RIGHT ANSWER is a for  
all 14 questions

Ayman Badawi

**QUESTION 1.** (i) Let  $T : R^2 \rightarrow R^2$  be a L. T such that  $T(2, 0) = (4, 6)$  and  $T(0, 3) = (6, 12)$ . The standard matrix presentation of  $T$ ,  $M$ , is

(a)  $M = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$       (b)  $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$       (c)  $M = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$       (d)  $M = \begin{bmatrix} 4 & 6 \\ 6 & 12 \end{bmatrix}$

(ii) Let  $T : R^2 \rightarrow R$  be a L. T such that  $T(2, 2) = 4$  and  $T(0, 3) = 6$ . Then  $T(1, 7) =$

(a) 14      (b) 10      (c) 15      (d) 13

(iii) Let  $T : R^3 \rightarrow R^2$  such that  $T(x_1, x_2, x_3) = (x_1 - 2x_3, -2x_1 + 4x_3)$ . The set of all points (vectors) in  $R^3$  where  $T(x_1, x_2, x_3) = (5, -10)$  is

(a)  $\{(5 + 2x_3, x_2, x_3) \mid x_2, x_3 \in R\}$       (b)  $\text{span}\{(5, 0, 0)\}$       (c)  $\text{span}\{(5, 0, 0), (2, 0, 1), (0, 1, 0)\}$   
(d)  $\{(5 + 2x_3, 0, x_3) \mid x_3 \in R\}$       (e)  $\text{span}\{(2, 0, 1), (0, 1, 0)\}$

(iv) Let  $T$  as in (iii). The  $\text{Ker}(T) =$

(a)  $\text{span}\{(2, 0, 1), (0, 1, 0)\}$       (b)  $\text{span}\{(0, 1, 0)\}$       (c)  $\{(2x_3, 0, x_3) \mid x_3 \in R\}$       (d)  $\text{span}\{(2, 1, 1)\}$

(v) One of the following is a linear transformation from  $R^3$  to  $R^2$

(a)  $T(x_1, x_2, x_3) = (-5x_1 + 10x_2, x_1 + 2x_3)$       (b)  $T(x_1, x_2, x_3) = (x_2, 4)$   
(c)  $T(x_1, x_2, x_3) = (0, x_1x_3)$       (d)  $T(x_1, x_2, x_3) = (2 + x_1, x_2)$       (e) (b) and (c)

(vi) Let  $D = \text{span}\{(1, -1, -1), (1, 0, 0), (3, -1, -1)\}$ . A basis for  $D$  is

(a)  $B = \{(1, -1, -1), (1, 0, 0)\}$       (b)  $B = \{(1, -1, -1), (1, 0, 0), (3, -1, -1)\}$   
(c)  $B = \{(1, -1, -1), (0, -1, 2)\}$       (d)  $B = \{(1, -1, -1)\}$       (e) (a) and (c)

(vii) Let  $D$  be as in (vi). One of the following points (vectors) belongs to  $D$ .

(a) (6, -3, -3)      (b) (5, 2, -2)      (c) (5, -4, 4)      (d) (10, 4, -4)

(viii) Let  $D = \{(a - 2b, 2a - 4b, a, b) \mid a, b \in R\}$ . Then  $\dim(D) =$

(a) 2      (b) 4      (c) 1      (d) 3

(ix) One of the following is a subspace of  $R^3$ ,

(a)  $D = \{(3b - c, b, c) \mid b, c \in R\}$       (b)  $\{(3bc, b, c) \mid b, c \in R\}$       (c)  $\{(3, b, c) \mid b, c \in R\}$   
(d)  $\{(3c, b, c) \mid b, c \geq 0\}$       (e) a, c

(x) Let  $D = \{(a - 2b + 3c, -2a + 4b - 6c, -a + 2b - 3c, 3a - 6b + 9c) \mid a, b, c \in R\}$  be a subspace of  $R^4$ . A basis for  $D$  is

(a)  $B = \{(1, -2, -1, 3)\}$

(b)  $B = \{(-2, 4, 2, -6), (3, -6, -3, 9)\}$

(c)  $B = \{(1, -2, -1, 3), (-2, 4, 2, -6), (3, -6, -3, 9)\}$

(xi) Let  $D$  as in (x). One of the following point (vector) belongs to  $D$ .

(a)  $(-6, 12, 6, -18)$

(b)  $(-1, -2, 1, -3)$

(c)  $(-3, -6, 3, -9)$

(d)  $(-4, 8, 4, 12)$

(xii) Let  $A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 5 & 6 & 6 & 7 \end{bmatrix}$ ,  $B = \begin{bmatrix} 7 & 8 & 0 & 4 \\ 5 & 1 & 1 & 8 \\ 7 & 1 & -1 & 1 \\ -9 & 9 & 0 & 3 \end{bmatrix}$  and  $AB = C$ . The third column of  $C$ , i.e.,  $C_3$  is

(a)  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b)  $\begin{bmatrix} 4 \\ 12 \end{bmatrix}$

(c)  $\begin{bmatrix} 0 \\ 0 \\ 2 \\ 6 \end{bmatrix}$

(d)  $\begin{bmatrix} 0 \\ 0 \\ 4 \\ 12 \end{bmatrix}$

(xiii) Let  $Q_1, Q_2, Q_3, Q_4$  be independent points (vectors) in  $R^4$ . Then

(a) The only solution to  $c_1Q_1 + c_2Q_2 + c_3Q_3 + c_4Q_4 = (0, 0, 0, 0)$  is  $c_1 = c_2 = c_3 = c_4 = 0$ .

(b) It is possible that  $Q_1 = c_2Q_2 + c_3Q_3$  for some  $c_1, c_2 \in R$ .

(c) It is possible that  $\text{span}\{Q_1, Q_2, Q_3, Q_4\} \neq R^4$ .

(xiv) The solution set to the homogeneous system

$$x_2 - x_3 - 4x_4 = 0, \quad x_1 - x_2 - 3x_3 - x_4 = 0, \quad 2x_2 - 2x_3 - 8x_4 = 0$$

(a)  $\text{span}\{(4, 1, 1, 0), (5, 4, 0, 1)\}$       (b)  $\{(-4x_3 - 5x_4, -x_3 - 4x_4, x_3, x_4) \mid x_3, x_4 \in R\}$

(c)  $\text{span}\{(5, 4, 0, 1)\}$       (d)  $\{(-4x_3 - 5x_4, x_3 + 4x_4, x_3, x_4) \mid x_3, x_4 \in R\}$