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MTH 418, Graph Theory, Exam Three

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$\frac{65}{65}$

QUESTION 1. (15 points) Let $G = K_{3,2}$

(i) Is G an Eulerian trail? If yes, then construct such trail.

all edges → $v_0 \neq w_0$

only ~~2 v~~ 2 vertices with ~~g~~ degree = odd

~~1-4-2-5-3-4~~

4 – 1 – 5 – 2 – 4 – 3 – 5

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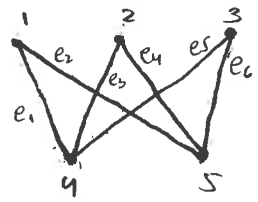

all vertices

(ii) Is G a Hamiltonian path? If yes, then construct such trail.

1 – 4 – 2 – 5 – 3

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(iii) Find the chromatic number of G →

$\chi(G) = 2$

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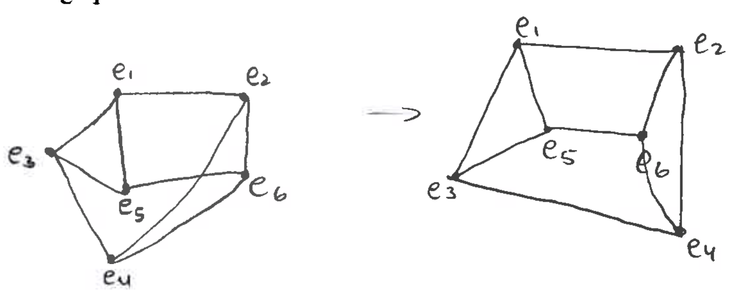
(iv) Draw the line graph of G.



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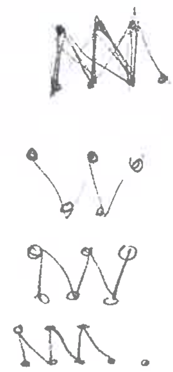

(v) Find the Edge-chromatic number of G and the chromotic number of L(G).

$\chi'(G) = 3$ $\chi(L(G)) = 3$

5

$3+4-5=2$

QUESTION 2. (24 points

(i) For what values of m is $K_{6,m}$ a Hamiltonian path?

all vertices

$m = 5$ or 6 or 7

5

(ii) Prove that K_5 is not a planar. Is K_8 a planar?

$\to$ Girth = 3

$|E| = \frac{5 \cdot 4}{2} = 10$

∴ K_5 is not planer.

∮ planer: $|F| + |V| - |E| = 2$

$|F| = 2 - |V| + |E|$

$3|F| \leq 2|E|$

$3(2 - |V| + |E|) \leq 2|E|$

$6 - 3|V| + 3|E| \leq 2|E|$

$|E| \leq 3|V| - 6$

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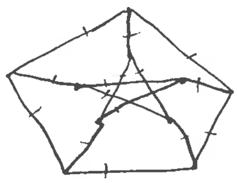
Since K_5 is a subgraph of K_8, K_8 is not planer.

(iii) Prove that Peterson graph is not a planar.

$10 \leq 15 - 6 \to 10 \nleq 9$ contradiction



DENY. ASSUME PETERSEN IS PLANER $\to$ Girth = 5, $|V| = 10$, $|E| = 15$

$|F| + |V| - |E| = 2$

$|F| = 2 + |E| - |V|$

$5|F| \leq 2|E|$

5

$5(2 + |E| - |V|) \leq 2|E| \to 10 + 5|E| - 5|V| \leq 2|E|$

$3|E| \leq 5|V| - 10$

$|E| = 15$

$|V| = 10$

$45 \leq 40 \to 45 \nleq 40$

contradiction

∴ petersen is not planer

(iv) prove that Q_4, is not a planar. Is Q_9 a planar?

$\to Q_9$ is not planer as Q_4 is subgraph of Q_9 and Q_4 is not planer.

$|V| = 16$

$|E| = 32$

5

① $F = 2 + |E| - |V|$

Girth = 4

② $4|F| \leq 2|E|$

① & ②

$4(2 + |E| - |V|) \leq 2|E|$

$8 + 4|E| - 4|V| \leq 2|E|$

$|E| \leq 2|V| - 4$

$32 \leq 28$

contradiction

Hence Q_4 is not planer.

(v) What is the chromatic number of Q_5? prove your claim.

$\chi(Q_5) = 2$ because Q_5 is a connected bipartite graph

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(vi) Find the dominating number of $Q3$ and construct such set

$\gamma(Q_3) = 2$

$D = \{000, 111\}$

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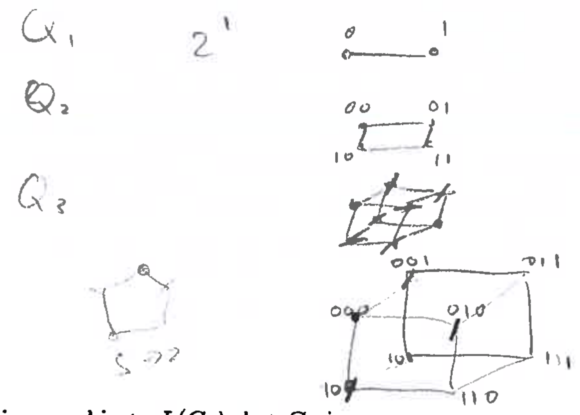
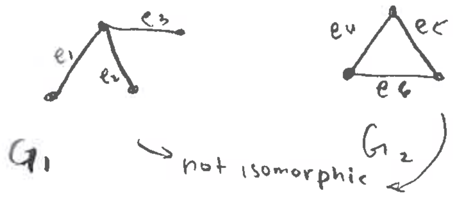


(vii) give me an example of two graphs G_1, G_2 such that $L(G_1)$ is graph-isomorphic to $L(G_2)$, but G_1 is not graph-isomorphic to G_2.



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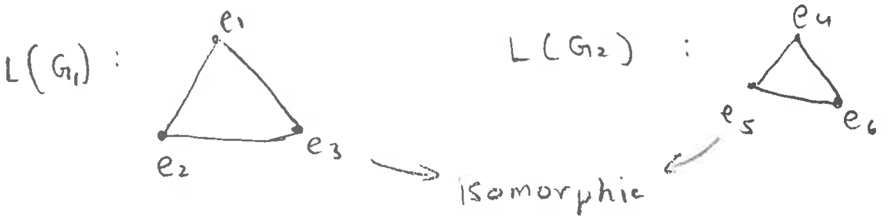

(viii) Find the degree of each vertex of $L(K_9)$ How many edges does $L(G)$ have?

$e \in V(L(K_9))$ $\deg(e) = \deg(a) + \deg(b) - 2$ $\quad \forall e \in L(K_9)$

$= 8 + 8 - 2$

$\deg(v) = 14 \quad \forall v \in V(L(K_9))$

$|E| = \dfrac{8^2 \cdot 9 - 2\left(\frac{9 \cdot 8}{2}\right)}{2} = 252$

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Faculty information

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$\frac{4 \cdot 3}{2} = 6$