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Exam III

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QUESTION 1. (6 points)

Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 7 & 1 & 8 & 4 & 5 & 2 \end{pmatrix}$

(a) Is f even or odd?

$f = (1\ 3\ 7\ 5\ 8\ 2\ 6\ 4)$

$= (1\ 4)(1\ 6)(1\ 2)(1\ 8)(1\ 5)(1\ 7)(1\ 3)$

$=$ 7 2-cycles so f is odd

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(b) Find $|f|$.

$f = (1\ 3\ 7\ 5\ 8\ 2\ 6\ 4)$

$|f| =$ length of cycle (just one cycle here)

$= 8$

QUESTION 2. (6 points)

Let $f = (1\,4\,7\,9) \circ (2\,3\,9\,6\,4)$ ← not disjoint!

1) Find |f|

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 2 & 3 & 7 & 5 & 6 & 9 & 8 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 3 & 9 & 2 & 5 & 4 & 7 & 8 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 3 & 1 & 2 & 5 & 7 & 9 & 8 & 6 \end{pmatrix}$$

$= (1\ 4\ 2\ 3) \circ (6\ 7\ 9)$ ← disjoint now!

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$|f| = \mathrm{lcm}(|(1\ 4\ 2\ 3)|, |(6\ 7\ 9)|)$

$= \mathrm{lcm}(4, 3) = 12$

QUESTION 3. (4 points) Convince me that A_{15} has an abelian subgroup of order 15.

We want an element of order 15 in A_{15}, say $f = \alpha_1 \circ \alpha_2$,
$\alpha_1 \in A_{15}$
$\alpha_2 \in A_{15}$

$|f| = 15 \Rightarrow \operatorname{lcm}(\underset{3}{|\alpha_1|}, \underset{5}{|\alpha_2|}) = 15$

say $\alpha_1 = (1\ 3\ 5)$
$\alpha_2 = (2\ 4\ 6\ 8\ 10)$

$f = (1\ 5)(1\ 3)(2\ 10)(2\ 8)(2\ 6)(2\ 4)$
so f is even

$f = (1\ 3\ 5) \circ (2\ 4\ 6\ 8\ 10)$

An element of order 15 will form a subgroup of 15 elements, let's call this subgroup H

$\boxtimes$ cyclic groups are Abelian

$|H| = 15$ & $H = \langle f \rangle$ so H is cyclic $\Rightarrow$ H is Abelian

so A_{15} has an Abelian subgroup of order 15

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QUESTION 4. (6 points) Up to isomorphism, classify all abelian group of order 50.

$50 = 2 \cdot 25 = 2 \cdot 5^2$ (prime factorization)

2	5^2
$\mathbb{Z}_2$	$\mathbb{Z}_{5^2} = \mathbb{Z}_{25}$
	$\mathbb{Z}_5 \times \mathbb{Z}_5$

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Options are $\mathbb{Z}_2 \times \mathbb{Z}_{25}$ and $\mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_5$

So for any Abelian group of order 50, say G, $G \approx \mathbb{Z}_2 \times \mathbb{Z}_{25}$
$G \approx \mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_5$

Faculty information

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