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TR section

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Final Exam, MW

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QUESTION 1. (i) **(3 points)** Let $A = \{1, 2, 6, 12, 3, 15\}$. Define "<=" on A such that a" $\leq$ "b iff $b \mid a$. Then " <= " is a partial order relation on ~~Z~~ A

b is a Factor of a

Write down T or F

a. $\overset{a}{6} \leq \overset{b}{12}$ ~~T~~ F is 12 Factor of 6

b. $\overset{a}{15} \leq \overset{b}{3}$ ~~F~~ T is 3 Factor of 15

c. $\overset{a}{6} \leq \overset{b}{1}$ ~~F~~ T is 1 Factor of 6

QUESTION 2. (i) **(8 points)** Let $a_n = 7a_{n-1} - 12a_{n-2} + 40n$. Find a general formula for a_n. Do not find C_1, C_2.

$40n \to an+b$

$a_n - 7a_{n-1} + 12a_{n-2} = 40n$

$\alpha^2 - 7\alpha + 12 = 40n$

$\alpha = 4 \quad \alpha = 3$

$a_n = C_1 \cdot 4^n + C_2 3^n$

$an+b - 7(a(n-1)+b) + 12(a(n-2)+b) = 40n$

$an+b - 7(an-a+b) + 12(an-2a+b) = 40n$

$an+b - 7an + 7a - 7b + 12an - 24a + 12b = 40n$

$\underset{6an}{an - 7an + 12an} + \underset{-17a}{7a - 24a} + \underset{6b}{b - 7b + 12b} = 40n$

$6an - 17a + 6b = 40n$

$6a = 40$

$a = \frac{20}{3}$

$6b - 17a = 0$

$6b = 17a$

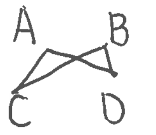
$6b = \frac{340}{3}$

$b = \frac{170}{9}$

$a_n = 4^n c_1 + 3^n c_2 + \frac{20}{3}n + \frac{170}{9}$

(ii) **(3 points)** Is $K_{2,2}$ an Eulerian circuit ? If yes, construct such circuit.

yes



deg of vertex is even for all values

QUESTION 3. (6 points)

(i) **(3 points)** The digits 0, 1, 2, ..., 8 will be used to construct cars plates. How many car-plates can be constructed if the second digit must be even, the third digit must be 7, the 4th digit must be 6, and the last digit must be even ? Note that each plate consists of 6 digits and there is no repetition.

5 even
4 odd

		odd	even		even
5	4	1	1	4	3

$2+3=5$

$5 \times 4 \times 1 \times 1 \times 4 \times 3$

$= 240$

(ii) **(3 points)** There are 1007 positive integers, where each is of the form $10k$ for some integer k. Then there are at least m integers out of the given 1007 integers say $a_1, \ldots, a_m$ such that $a_1 \ (mod\ 8) = a_2 \ (mod\ 8) = \cdots = a_m \ (mod\ 8)$. What is the best value of m? show the work

$0 \to 0$
10 mod 8 = 2
20 mod 8 = 4
30 mod 8 = 6
40 mod 8 = 0

50 mod 8 = 2
60 mod 8 = 4
70 mod 8 = 6

$(0, 2, 4, 6) \to 4$

$\frac{1007}{4} = 252$

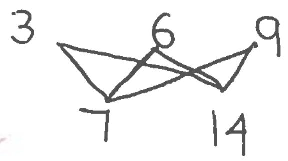
QUESTION 4. Let $V = \{3, 6, 7, 9, 14\}$. Two vertices $v_1, v_2 \in V$ are connected by an edge if and only if $ab \ (mod\ 21) = 0$.

(i) **(4 points)** By drawing the graph, convince me that the graph is a $K_{m,n}$ for some integers m, n.

$3 \times 7 = 21$

$3 \to 6, 9, 3$

$7 \to 7, 14$



$K_{3,2}$

$m = 3$
$n = 2$

(ii) **(4 points)** Is the graph an Eulerian Trail? If yes, construct such trail.

$7 \to 3 \to 14 \to 6 \to 7 \to 9 \to 14$

yes eulerian since odd degree of vertex for 7, 14

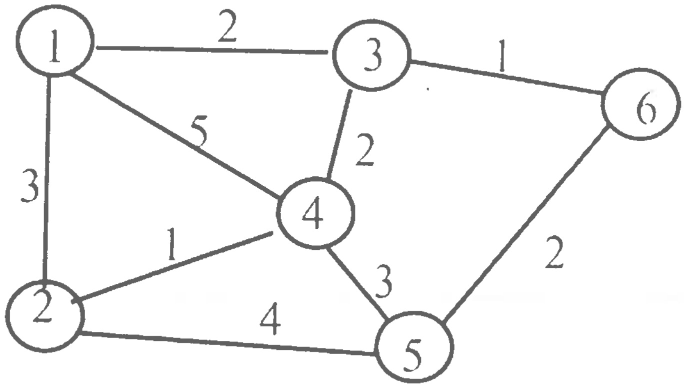
(iii) **(3 points)** Convince me that the graph is not a Hamiltonian.

the graph is bipartite & C_5 is not a subgraph, therefore the graph is not Hamiltonian

QUESTION 5. (8 points)

Stare at the below picture.

Consider the network (nodes, links and their weights) in the figure below.



Use Dijkstra's Algorithm to construct the minimum weight spanning tree from vertex 1 to every other vertex.

V	1	2	3	4	5	6
1	$\boxed{0}$	3^1	2^1	5^1	∞	∞
3		∞ ↓ 3^1	$\boxed{2^1}$	$2+2=4$ 4^3	∞	$1+2=3$ 3^3
2		$\boxed{3^1}$		$1+3$ 4^2	$4+3$ 7^2	∞ ↓ 3^3
6				∞ ↓ 4^2	$2+3=5$ 5^6	$\boxed{3^3}$
~~6~~4				$\boxed{4^2}$	$3+4$ ↓ 7 5^6	
5					$\boxed{5^6}$	

3 — 6 — 5

1

2 — 4

QUESTION 6. (6 points) Use Math Induction and prove that $24 \mid [(n+1)(n+3)(n+5)]$ for every odd integer $n \geq 1$.

1] prove For $n = 1$

$24 \mid (2)(4)(6)$ $\quad 48 = (24)(k)$ $\quad 48 = (24)(2)$

$k = 2$

2] assume $24 \mid (n+1)(n+3)(n+5)$ true For some $n \geq 1$

For $n+1$ $\quad n+2$

$24 \mid (n+2)(n+4)(n+6)$

$$\underbrace{(n+2)(n+4)(n+6)}_{\text{Factor of 24 by \#2}} + \underbrace{(n+1)(n+3)(n+5)}_{\text{Factor of 24 by \#1 \& by looking}}$$

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QUESTION 7. (4 points) Use the 4th method and prove that $\sqrt{29}$ is an irrational integer.

$\sqrt{29} = \frac{a}{b}$ $\quad a^2, b^2 \in \mathbb{Z},\ b \neq 0$ $\quad 29 \to \text{odd}$

$29 = \frac{a^2}{b^2}$ $\quad a = 2n+1$ $\quad m, n \in \mathbb{Z}$ $\quad \text{odd} = \frac{\text{odd}}{\text{odd}}$

$b = 2m+1$

$29 = \frac{(2n+1)^2}{(2m+1)^2}$

$29 = \frac{4n^2+4n+1}{4m^2+4m+1}$

$29(4m^2+4m+1) = 4n^2+4n+1$

$29.4m^2 + 29.4m + 29 = 4n^2+4n+1$

$\frac{29.4m^2 + 29.4m + 28}{4} = \frac{4n^2+4n}{4}$

$\underbrace{29m^2 + 29m + 7}_{\text{always odd}} = \underbrace{4n^2+4n}_{\text{not always odd}}$

$\text{odd} \neq \text{even}$

$\therefore \sqrt{29}$ is irrational

Faculty information

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