MTH 205, Summer 2021, 1-1

## Exam Two, MTH 205, Summer 2021

### Ayman Badawi

(Stop working at 14:45 pm/ submit your solution by 15:00 pm / DO NOT SUBMIT BY EMAIL) <sup>-</sup> 36

## **QUESTION 1.** (6 points)(SHOW THE WORK)

Find y(t), where  $y' - 7y = \int_0^t e^{2r+5t} dr - 12 \int_0^t y(r) dr$ , where y(0) = y'(0) = 0

### **OUESTION 2. (SHOW THE WORK)(6 points)**

Solve for x(t) and y(t) where

$$x^{(2)}(t) + y'(t) = 0$$
  
$$x'(t) - y(t) = -4$$

, given y(0) = 5, x(0) = 5, x'(0) = 1

**QUESTION 3.** (SHOW THE WORK)(6 points) Find  $y_g$ , where  $y^{(3)} - 4y^{(2)} = 2$ .

**QUESTION 4.** (SHOW THE WORK)(4 points) Find  $y_g$ ,  $y^{(2)} - 4y' + 5y = 0$ ,

**QUESTION 5.** (SHOW THE WORK)(4 points) Find  $y_q$ ,  $t^2y^{(2)} - 3ty' + 5y = 0$ , t > 0,

# QUESTION 6. (SHOW THE WORK)(6 points) Find $y_g$ , $\frac{y^{(2)}}{t^2} + 2\frac{y'}{t^3} = -\frac{\ln(t)}{t^2}$ , t > 0,

**QUESTION 7.** (6 points) ONLY Describe the general form of  $y_p$  for the following LDE (Hint: Laplace might be useful as explained in the class).

(1) 
$$y^{(2)} + 6y' + 7y = \sin(3t) + 2021e^{-t} + 3e^{6t}$$

(2) 
$$y^{(2)} - y' = 2te^t + 5t + 6$$

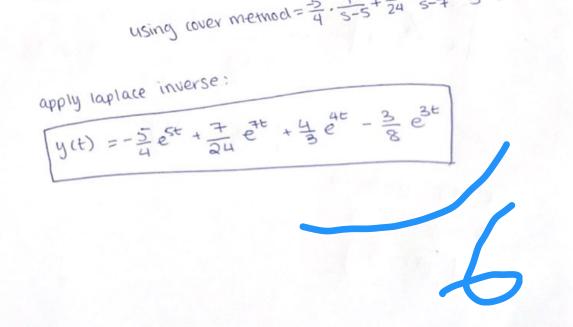
**QUESTION 8.** (6 points) Find  $y_q$ 

$$y^{(2)} + y = \frac{1}{\sin(t)}$$

#### **Faculty information**

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$$SY(S) - 7Y(S) + \frac{1}{5}Y(S) = \frac{1}{(S-5)(S-7)}$$

$$Y(S) [S - 7 + \frac{1}{5}] = \frac{1}{(S-5)(S-7)}$$

$$Y(S) [\frac{S^{2} - 7S + 12}{S}] = \frac{1}{(S-5)(S-7)}$$

$$Y(S) = \frac{S}{(S-5)(S-7)(S-4)(S-3)} = \frac{A}{S-5} + \frac{A}{S-7} + \frac{C}{S-4} + \frac{D}{S-3}$$

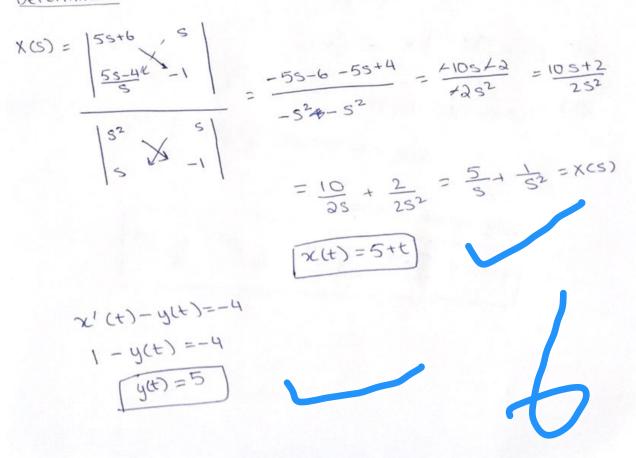
$$Y(S) = \frac{S}{(S-5)(S-7)(S-4)(S-3)} = \frac{A}{S-5} + \frac{1}{S-5} + \frac{1}{24} + \frac{1}{S-7} + \frac{3}{S-4} + \frac{3}{S-3} + \frac{1}{S-3}$$

$$USin (Giver method = \frac{5}{4} + \frac{1}{S-5} + \frac{1}{24} + \frac{1}{S-7} + \frac{3}{S-4} + \frac{3}{S-3} + \frac{1}{S-3}$$

Apply Laplace to  

$$SY(S) - Y(O) - 7Y(S) = L \frac{2}{5} e^{2t} \cdot e^{5t} dr \frac{2}{5} - \frac{12}{5} Y(S)$$
  
 $L \frac{2}{5} e^{5t} \cdot \int e^{2t} dr \frac{2}{5} = \frac{1}{5-5} \cdot \frac{1}{(5-5)-2} = \frac{1}{(5-5)(5-7)}$ 

$$\frac{Q1}{y'-7y} = \int_0^t e^{3r+5t} dr - i \partial_i \int_y^t y(r) dr \quad \text{given } y(0) = 0$$



Determinate

$$S^{2}X(S) + SY(S) = 5S + 6$$
  
 $SX(S) - Y(S) = \frac{5S - 4}{S}$ 

Simplify

Q2

Apply Laplace:

$$x''(t) + y'(t) = 0$$
  
 $x'(t) - y(t) = 0$ 

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$$\begin{array}{l} \mathcal{Y}_{\mathcal{Y}} = 0 \\ \hline & \mathcal{Y}_{\mathcal{Y}} = e^{2t} \left( \zeta_{1} \cos t + \zeta_{2} \sin t \right) \\ \mathcal{Q}_{\mathcal{Y}} = f^{2t} \left( \zeta_{1} \cos t + \zeta_{2} \sin t \right) \\ \mathcal{Q}_{\mathcal{Y}} = t^{2t} \left( cauchy = 3ty + 5y = 0 \\ y = t^{n} \left( cauchy = cuter - homogenous \right) \\ characteristic \quad polynomial : m^{2} - m - 3m + 5 = 0 \\ (by staring) \quad m^{2} - 4m + 5 = 0 \\ m = 2ti \\ \hline & \mathcal{Y}_{\mathcal{Y}} = t^{2} \left( \zeta_{1} \cos(\ln tt) + \zeta_{2} \sin(\ln(tt)) \right) \quad \text{where } t > 0 \\ \end{array}$$

y=e<sup>mt</sup> (nomogeneous) characteristic polynomial: m<sup>2</sup>-4m+5=0 m=2±i (use 1 result be imaginary)

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(9)

find  $y_{g} = \frac{y''}{t^{2}} + \frac{2y'}{t^{3}} = -\frac{\ln(t)}{t^{2}}$ 900089492 Hafsan Tani -> cauchy euler y=> m2-m+2m=0  $m^{2} + m = 0$ m(m+1) = 0 $m=0 \quad m=-1 \longrightarrow C_1 + C_2 t^{-1} = y_n$ yp == variation  $V_{1} + V_{2} + V_{2} = 0$   $V_{1} + V_{2} +$  $V_1' + V_2' t^{-1} = 0$  $\begin{vmatrix} 1 & t^{-1} \\ 0 & -t^{-2} \end{vmatrix} - t^{-2} - 0 = -t^{-2}$  $\begin{vmatrix} 0 & t^{-1} \\ -\ln(t) & -t^{-2} \end{vmatrix} = 0 + t^{-1} \ln(t) = \frac{\pi t^{-1} \ln(t)}{\pi t^{-2}} \frac{\pi t^{-1} \ln(t)}{\pi t^{-2}} \frac{\pi t^{-1} \ln(t)}{\pi t^{-2}} = -t \ln(t) = v'_{1}$ by parts: u=In(t) v====t<sup>2</sup> du===tdt dv=tdt -1/2 In(t)+2 +1/2 +2 dt  $V_1 = -\frac{1}{2} \ln(t) t^2 + \frac{1}{4} t^2$ the stift for the

see next page ->

6)

$$\begin{aligned} y_{p} &= V_{1}(1) + V_{2}(t^{-1}) \\ y_{p} &= -\frac{1}{2}\ln(t)t^{2} + \frac{1}{6}t^{2} + \frac{1}{3}\ln(t)t^{2} - \frac{1}{6}t^{2} \\ y_{p} &= \sum_{36}^{5} \frac{5}{36}t^{2} - \frac{1}{6}\ln(t)t^{2} \\ y_{g} &= y_{n} + y_{p} \\ \hline y_{g} &= c_{1} + c_{2}t^{-1} + \frac{5}{36}t^{2} - \frac{1}{6}\ln(t)t^{2} \\ \end{aligned}$$

$$V_2 = \frac{1}{3} \ln(t) t^3 - \frac{1}{3} \int t^2 dt$$
  
 $V_2 = \frac{1}{3} \ln(t) t^3 - \frac{1}{3} t^3$ 

by parts 
$$w$$
  
 $u = lnt$   
 $du = \frac{1}{2}$ 

where  $V = \frac{1}{3}t^{3}$  $dv = t^{2}dt$ 

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See # eq 1 (on previous page)

 $V_{2}^{\prime} = t^{2} ln(t)$ 

 $\int V_2 dt = \int t^2 \ln(t) dt$ 

 $\frac{4}{t^2} + \frac{\sqrt{2}}{t^2} = -\pi \ln(t)$ 

$$y_g = c_1 \cos t + c_2 \sin t - t \cos t + sint \cdot \ln|sint|$$

$$(ost ) + (the cost - 1 = cost + V2')$$

 $V_1 = -t$ 

$$\begin{vmatrix} 0 & sint \\ -\frac{1}{1} = -1 = V_{1}' \\ \frac{1}{sint} & cost \\ V_{1} = \int -1 dt \\ V_{1} = \int -1 dt \end{vmatrix}$$

$$y_p = D$$
 variation  
 $V'_1 cost + V'_2 sint = D$   
 $-V'_1 sint + V'_2 cost = \frac{1}{sint}$ 

$$y'' + y = \frac{1}{sint}$$
  

$$y_{h} = D \quad m^{2} + 1 = D$$
  

$$m = \pm i$$

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